WIREFRAME MODELING

LECTURE #6

MKS 537E – Intro to CAE

Three dimensional Modeling

In 3D computer graphics, 3D modeling is the process of developing a mathematical coordinate-based representation of any surface of an object (inanimate or living) in three dimensions via specialized software by manipulating edges, vertices, and polygons in a simulated 3D space.

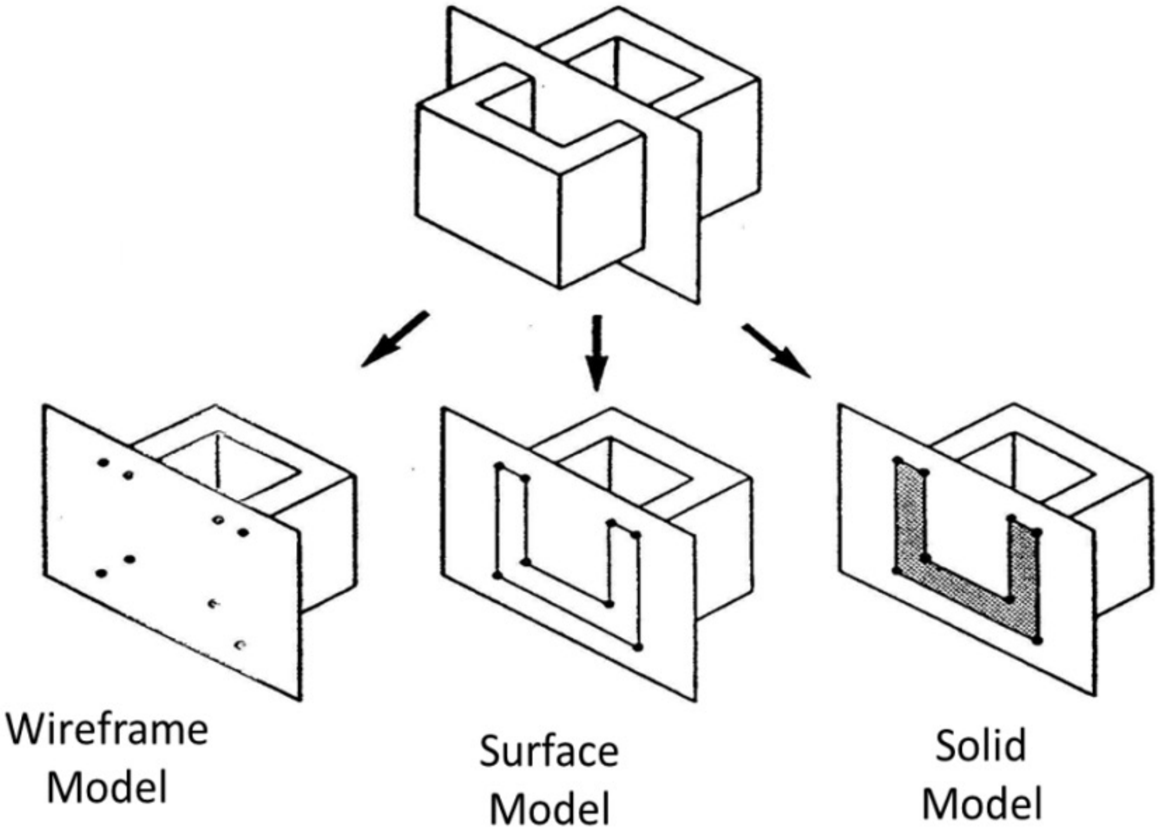
3D modeling

- 3D models are easier to interpret.
- Simulation under real-life conditions.
- Less expensive than building a physical model.
- 3D models can be used to perform finite element analysis (stress, deflection, thermal…..).
- 3D models can be used directly in manufacturing, Computer Numerical Control (CNC).
- 3D models can be used for presentations and marketing.

Geometric models

The three principal classifications can be

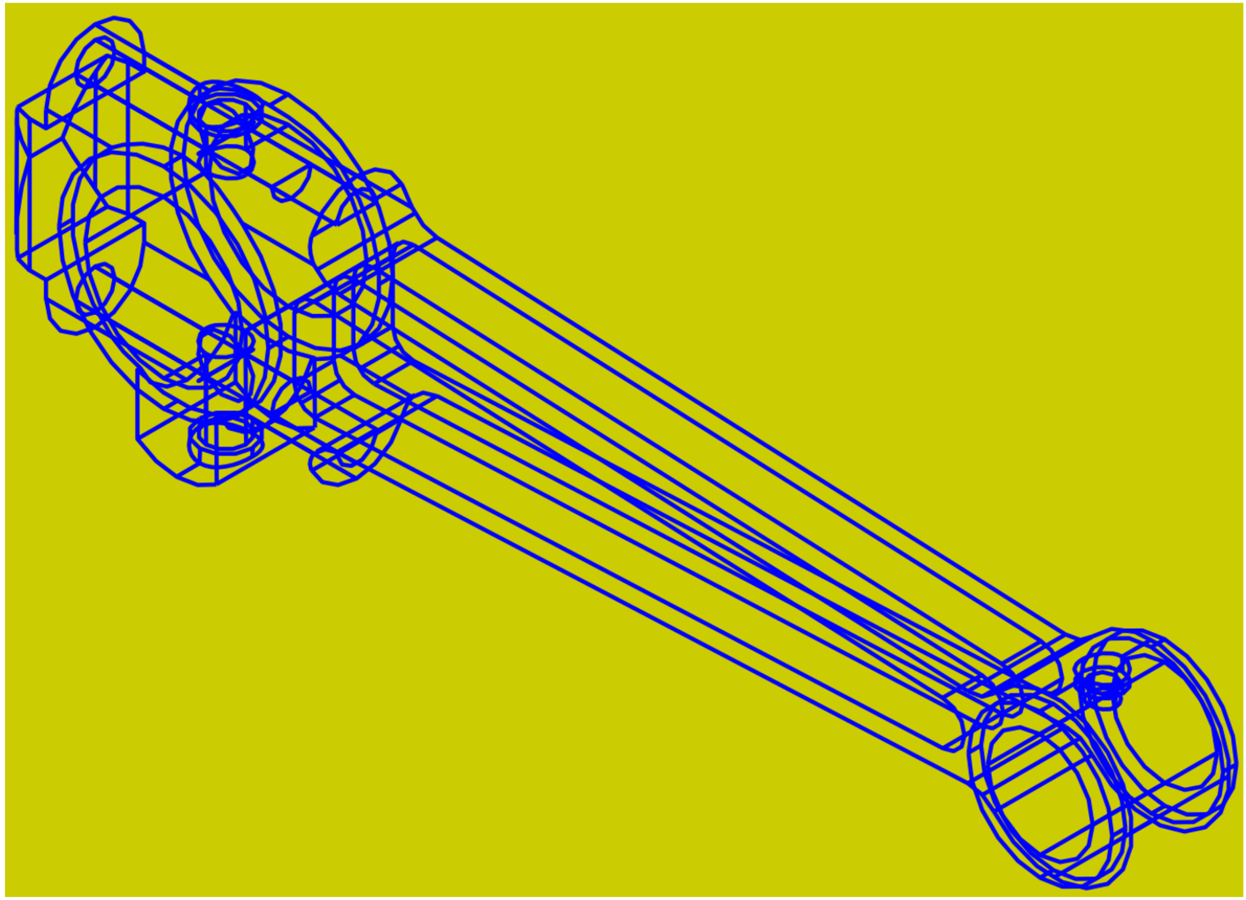
- ➢ The line model (wireframe model)
- ➢ The surface model
- ➢ The solid or volume model



What is wireframe modeling

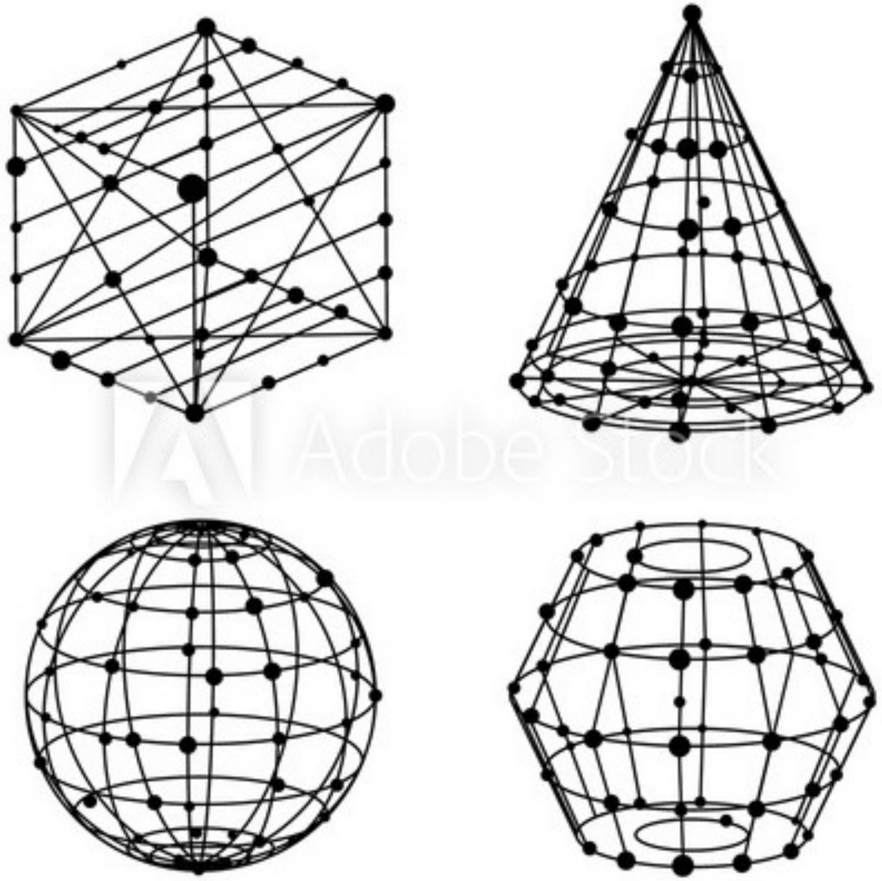
Wireframe modeling is the process of visual presentation of a three-dimensional or physical object used in 3-D computer graphics.

It is an abstract edge or skeletal representation of a real-world 3-D object using lines and curves.



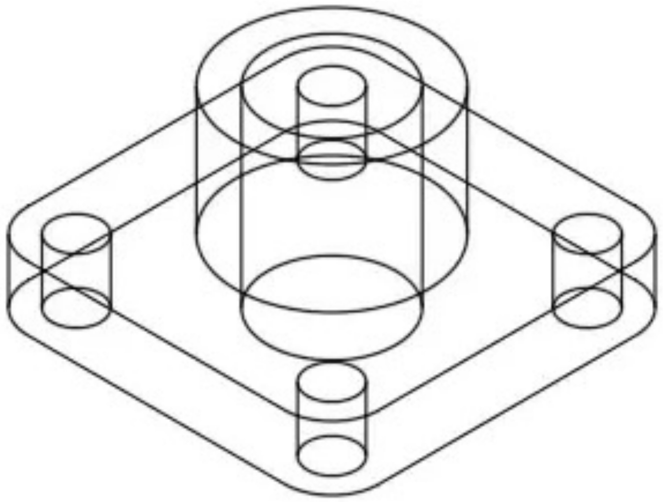
What is wireframe modeling

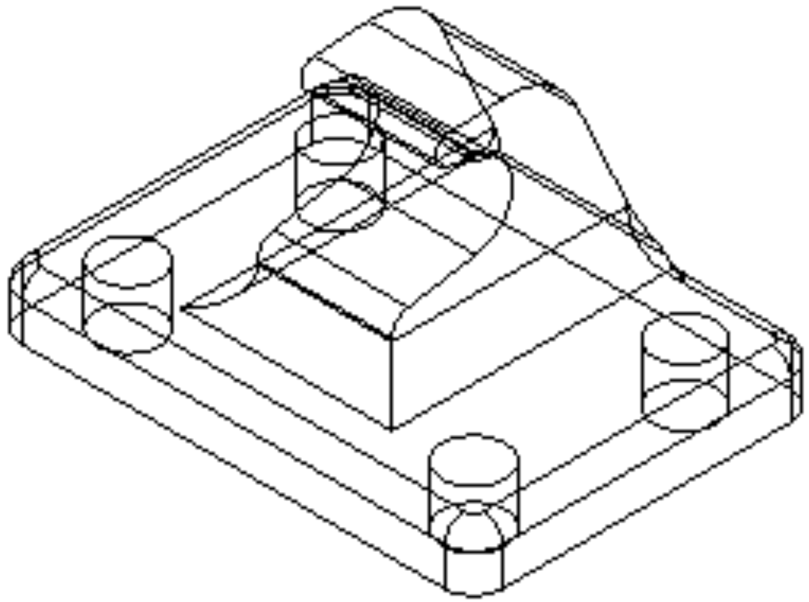
3D wire frame computer models allow for the construction and manipulation of solids and solid surfaces.
3D solid modeling efficiently draws higher quality representations of solids than conventional line drawing.



One can use a wire frame model to

- View the model from any vantage point
- Generate standard orthographic and auxiliary views automatically
- Generate exploded and perspective views easily
- Analyse spatial relationships, including the shortest distance between corners and edges, and checking for interferences
- Reduce the number of prototypes required



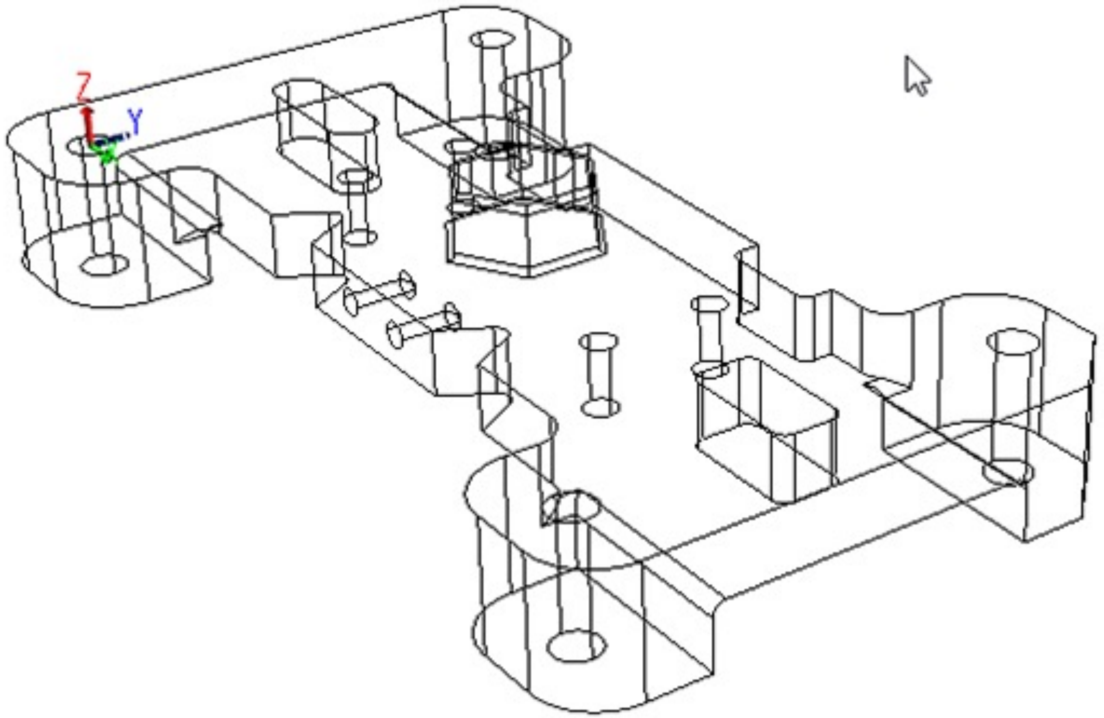


Wireframe modeling

There are two important aspects to the use of wire-frame models in CAD.

The first is the computer representation of an object, and this is concerned with the structure needed to encode a wire-frame model.

The second is concerned with the computational procedures needed to produce and manipulate the viewing or visualization of this representation.



Wireframe modeling

A computer representation of a wire-frame structure consists essentially of two types of information:

- The first is termed metric or geometric data which relate to the 3D coordinate positions of the wire-frame node’ points in space.
- The second is concerned with the connectivity or topological data, which relate pairs of points together as edges.

Wireframe modeling

Advantages

- Simple to construct
- Designer needs little training
- System needs little memory
- Take less manipulation time
- Retrieving and editing can be done easy
- Consumes less time
- Best suitable for manipulations as orthographic isometric and perspective views.

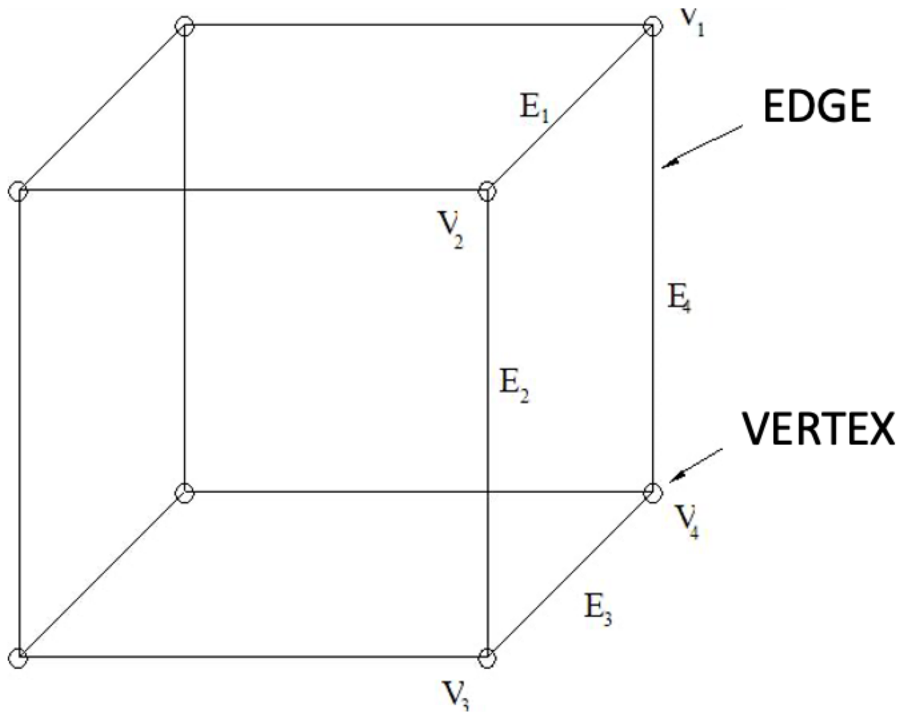
Disadvantages

- Image causes confusion
- Cannot get required information from this model
- Hidden line removal features not available
- Not possible for volume and mass calculation, NC programming cross sectioning etc
- Not suitable to represent complex solids

Wireframe model

The wireframe model is perhaps the oldest way of representing solids. Model is called a polygon net or polygon mesh.

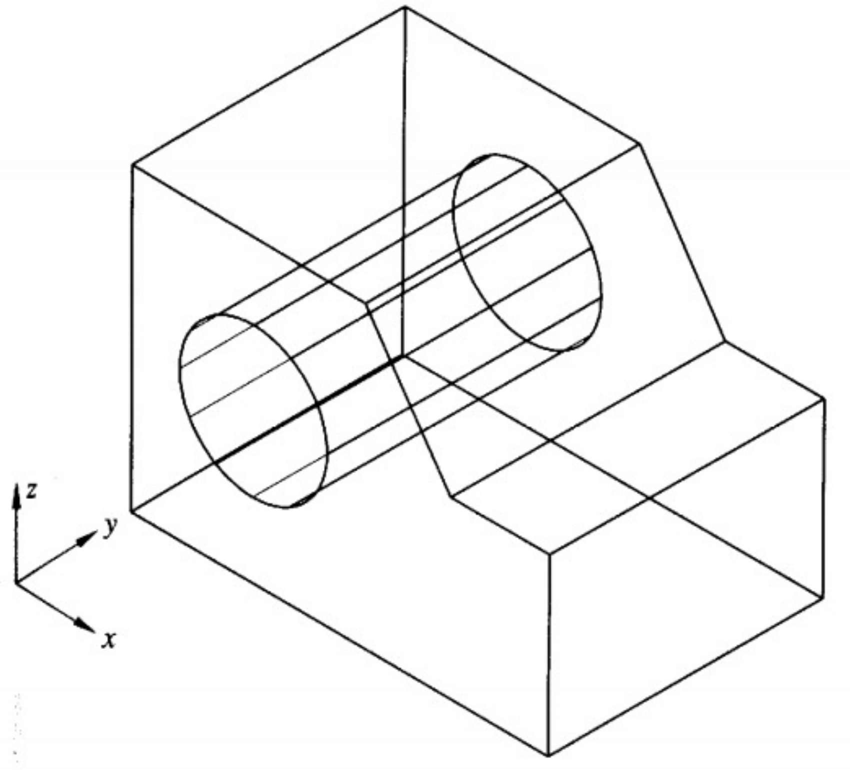
- Contains information about the locations of all the points (vertices) and edges in space coordinates.
- Each vertex is defined by x, y, z coordinate.
- Edges are defined by a pair of vertices.
- Faces are defined as three or more edges.
- Wireframe is a collection of edges, there is no skin defining the area between the edges.

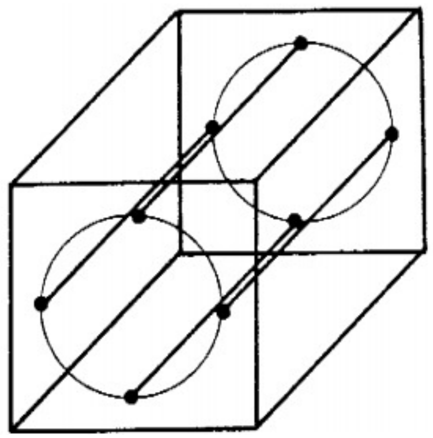


Wireframe model

A wire frame is effectively a line drawing of a 3-D object and is a method of representing the 3-D geometry of the edges and nodes of an object without a full surface representation.

The object has the appearance of a frame constructed from wire. It can be quickly displayed and manipulated. The block is shown as a wire frame with only 16 lines and 2 circles.

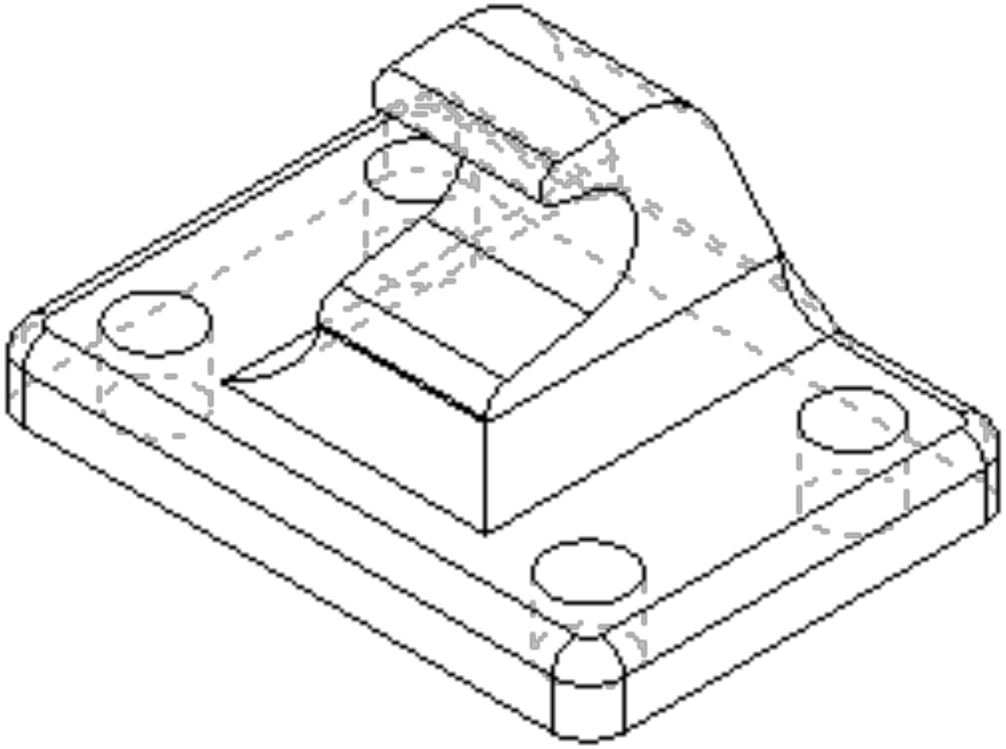




Hidden lines

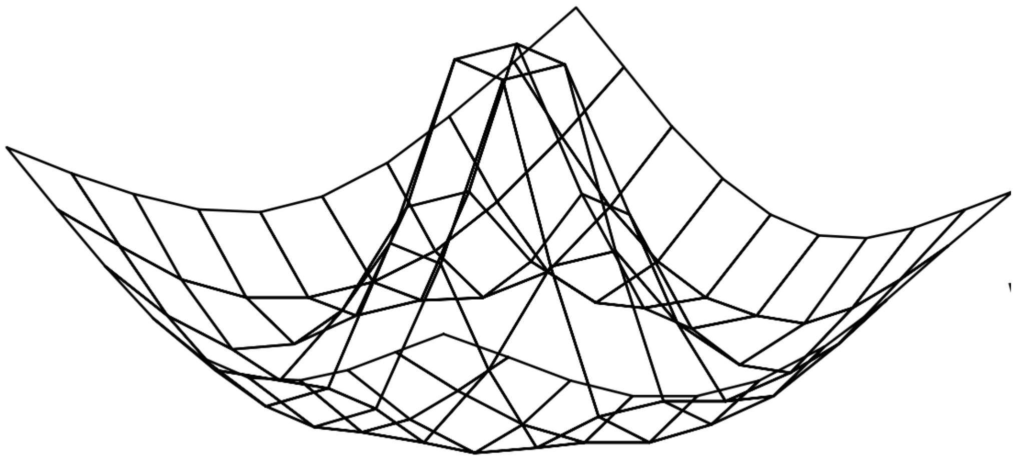
The difficulty with a wire frame model is that hidden lines are not removed and, for complex items, the result can be a jumble of lines that is impossible to determine.

Similarly, because surface features are not displayed there are no contour lines and so the surface can be ambiguous and the resulting interpretation of the object open to question.

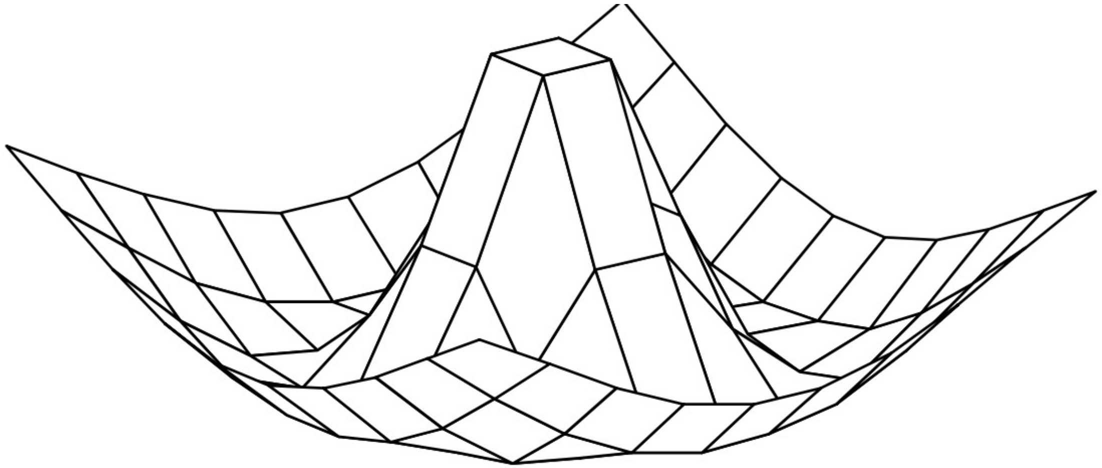


Hidden line removal

Without hidden lines removal



With hidden lines removal



Hidden line removal

Hidden line removal (HRL) is an extension of wire frame model rendering wher lines (or segments of lines) covered by surface are not drawn.

Hidden line removal is the method of computing whtich edges are not hidden by the faces of parts for a specified view and the display of parts in the projection of a model into a 2D plane.

It is considered that information openly exists to define a 2D wireframe model as well as the 3D topological information.

Hidden line algorithms

- Priority algorithm
- Area oriented algorithm
- Overlay algorithm

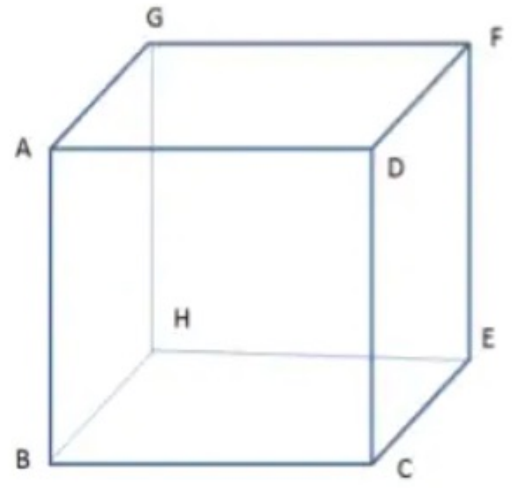
Priority algorithm

ABCD, ADFG, DCEF are given higher priority-1.

Hence all ,lines in this face are visible, that is, AB, BC, CD, DA, AD, DF, FG, AG, DC, CE, EF, and DF are visible.

AGHB, EFGH, BCEH are given lower priority-2.

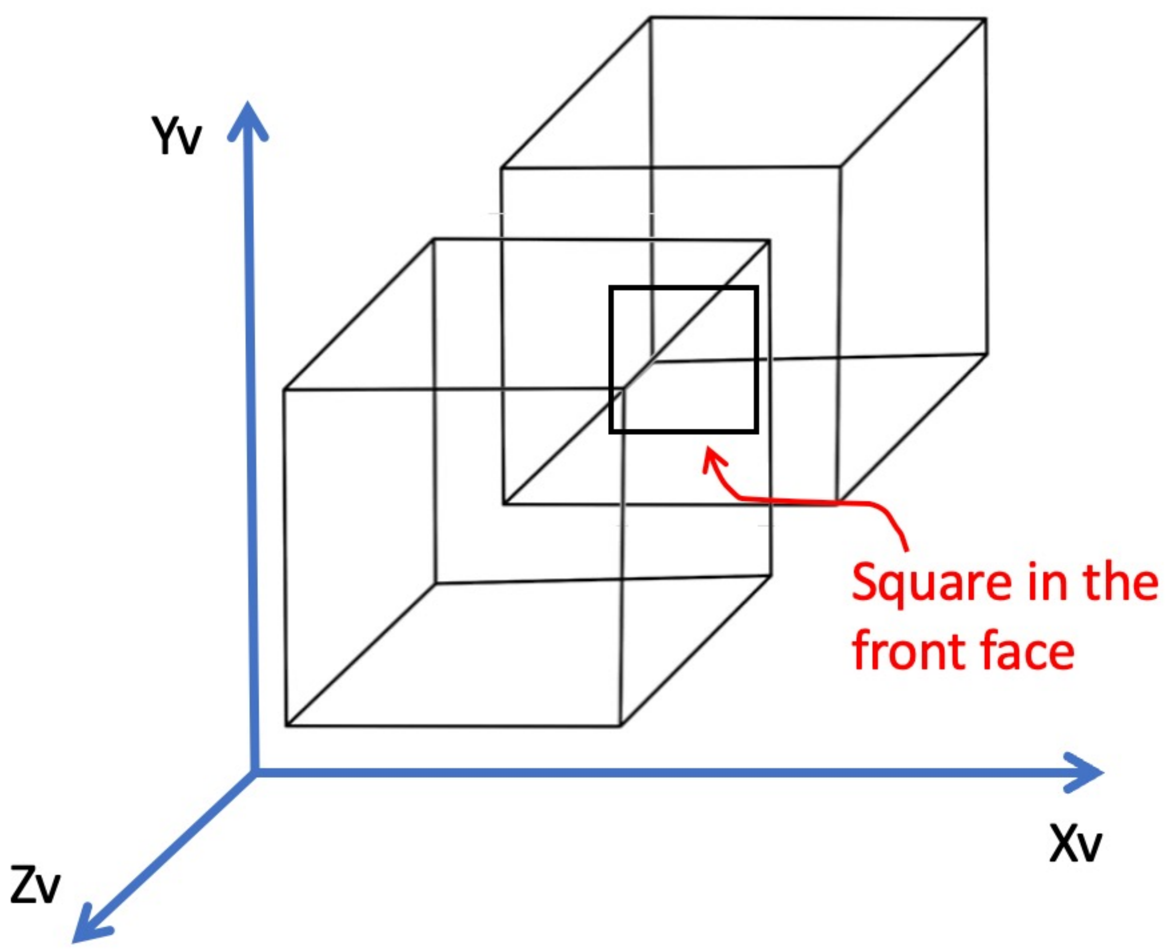
hence, all line this face other than priority-1 are invisible, that is BH, EH and GH. These lines must be eliminated.



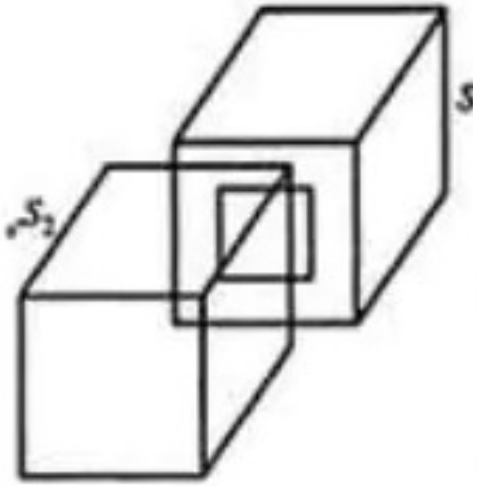
Face	Priority
ABCD	1
ADFG	1
DCEF	1
ABHG	2
EFGH	2
BCEH	2

Area oriented algorithm

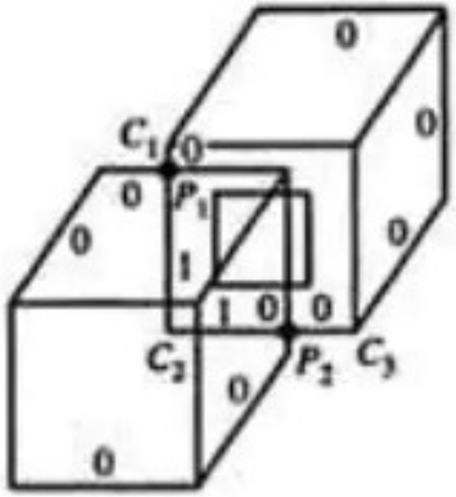
- Identiy silhouette polygons
- Quantitatice hiding (QH)
- Visibility of silhouette segment
- intersect the internal edges
- Display the edges



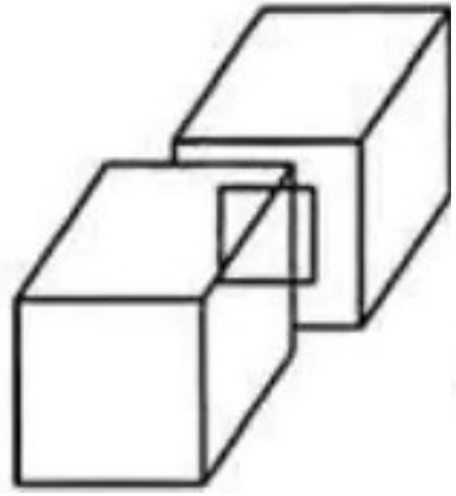
Area oriented algorithm



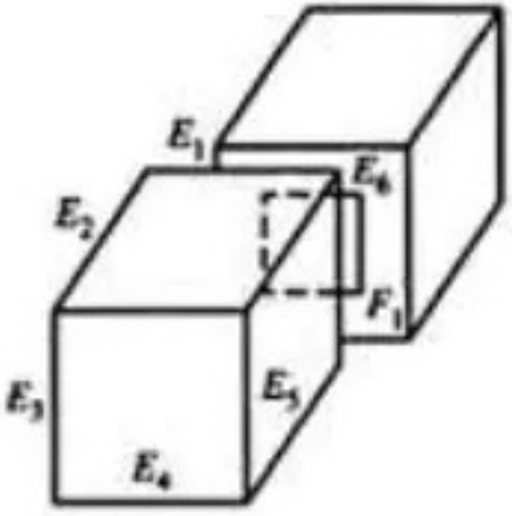
STEP 1
Silhouette
polygon



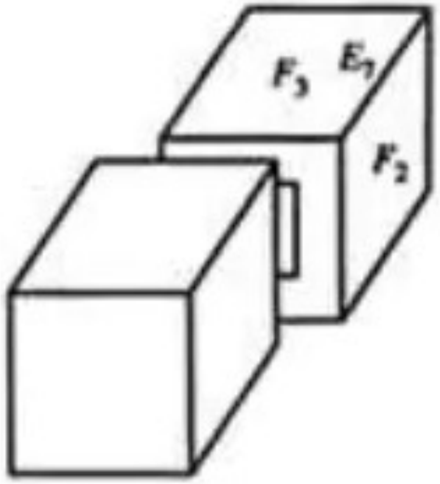
STEP 2
Quantitative
hiding



STEP 3
Visibility of
silhouette
segments



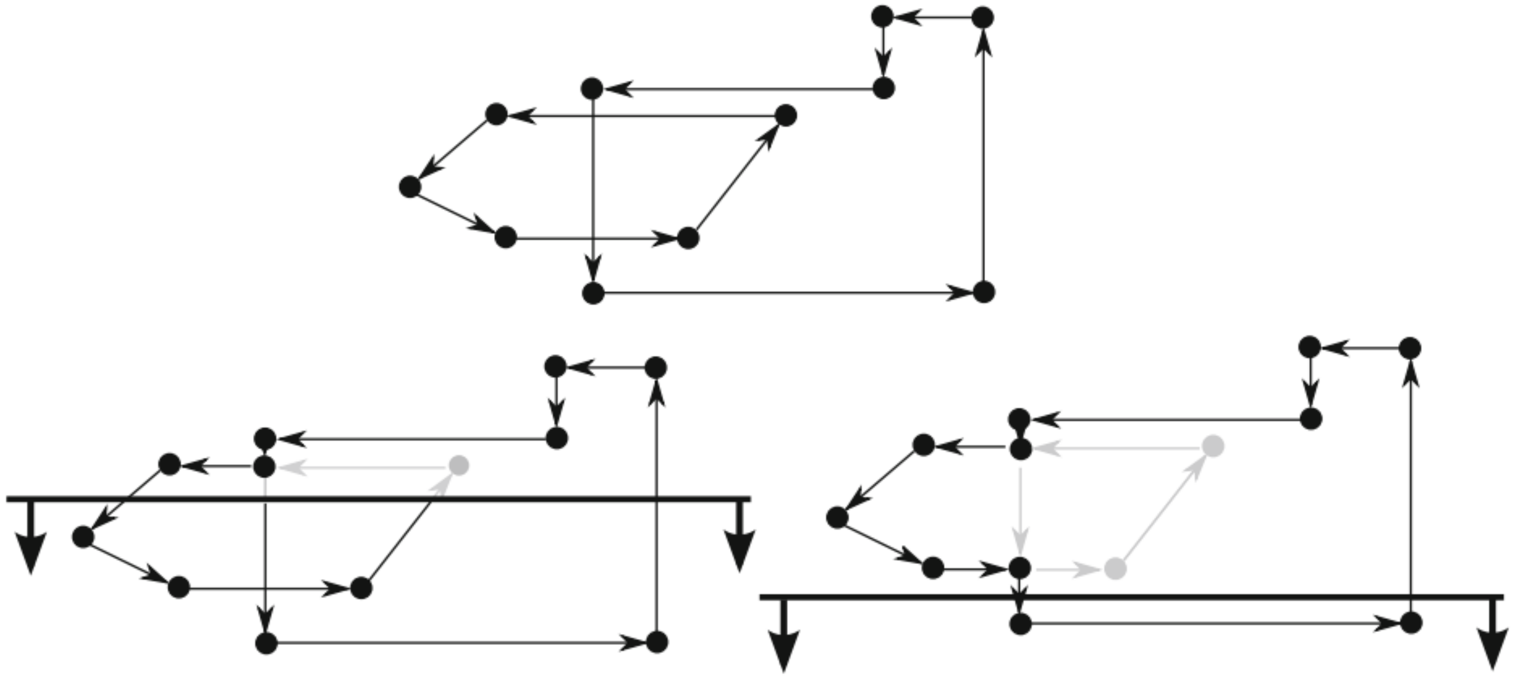
STEP 4
Visibility of
internal
edges



STEP 5
Display of
visible areas

Overlay algorithm

- Acceptable for curved surface by approximating them as plannar surfaces.
- U-v grid is used to make grid surface by making it as straight edges.



Basic database structures

- Relational database
  - A set of lists, uses arrays for storage
- Hierarchical database
  - A trees structure, think of a company's executive structure
- Network database
  - Use data pointers

Vertex list	Edge list
V1(0,0,0)	E1[V1,V2]
V2(1,0,0)	E2[V2,V3]
V3(0,1,0)	E3[V3,V1]
V4(0,0,1)	E4[V2,V4]
	E5[V4,V3]
	E6[V1,V4]

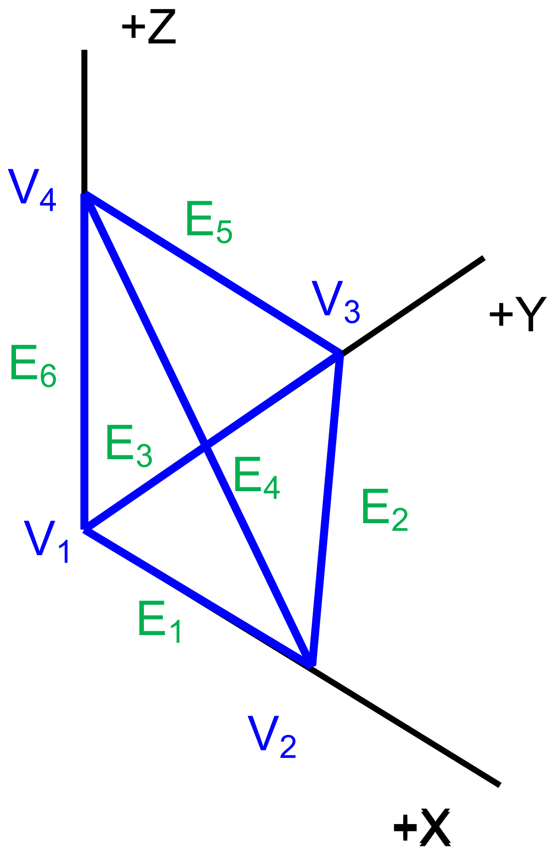
Rational database

A wireframe model consists of two tables, the *vertex* table and the *edge* table.

Each entry of the vertex table records a vertex and its coordinate values, while each entry of the edge table has two components giving the two incident vertices of that edge. A wireframe model *does not* have face information.

Rational database

- Vertices
  - Coordinate values for vertices
- Edges
  - Vertices associated with edges (endpoints)
- Faces
  - Loops (faces) formed by edges

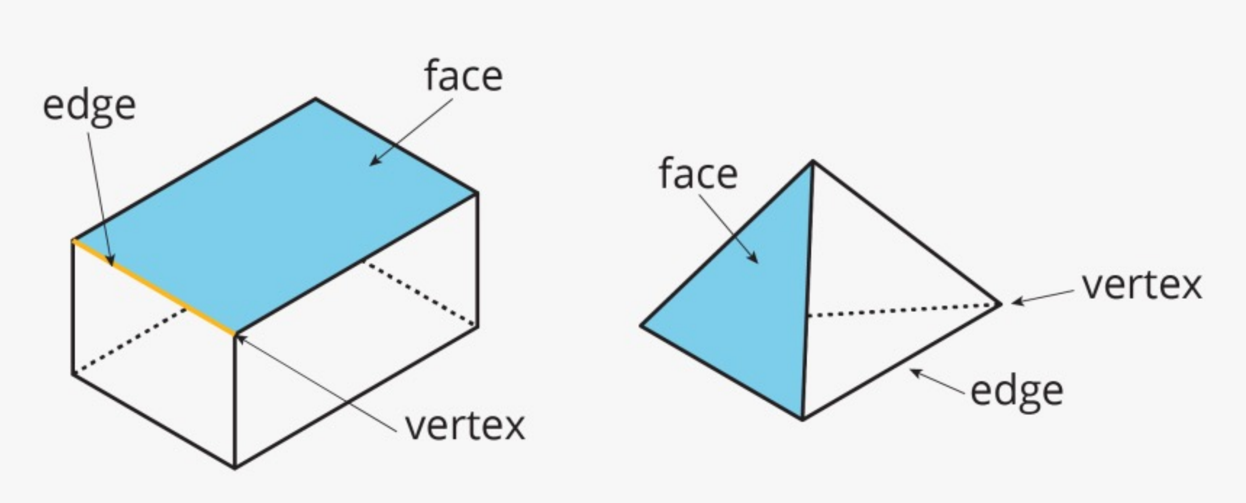


Wireframe tetrehedron

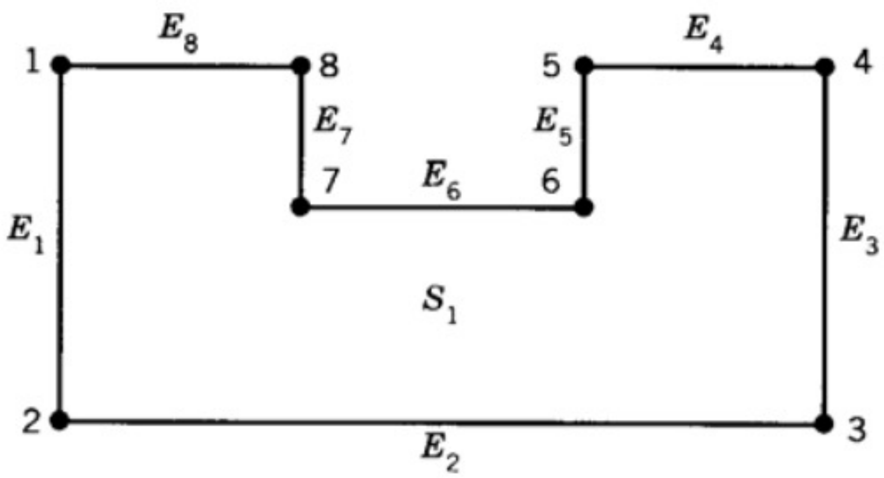
Vertex-edge-face graph

Typically, this hierarchy comprises the elements Body, Face, Edge and Vertex.

Each element is described by elements from the level beneath, i.e. the body is described by its faces, each face by its edges, each edge by a start and end vertex.



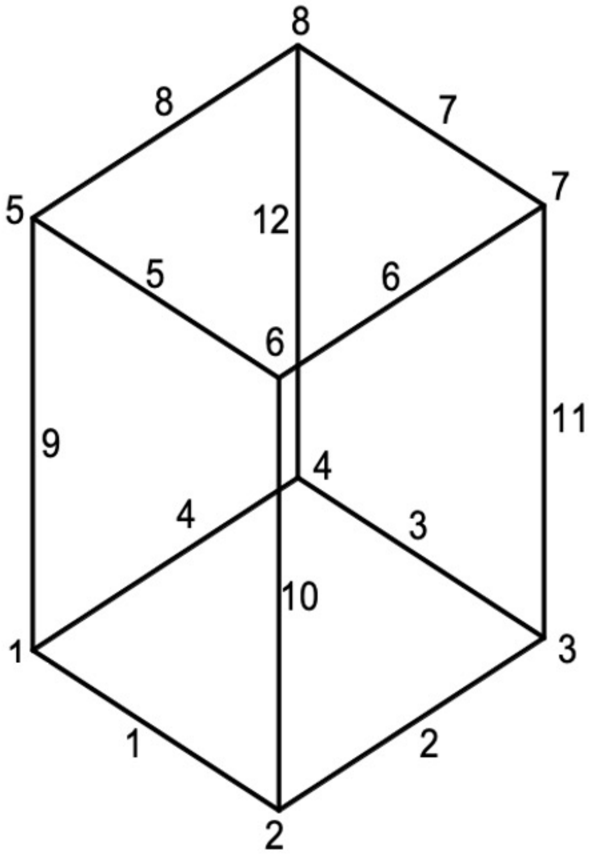
Rational database



Point	x	y	Line	Beginning point	Ending point	Surface	Line
1	x_1	y_1	E_1	1	2	S_1	E_1
2	x_2	y_2	E_2	2	3		E_2
3	x_3	y_3	E_3	3	4		E_3
4	x_4	y_4	E_4	4	5		E_4
5	x_5	y_5	E_5	5	6		E_5
6	x_6	y_6	E_6	6	7		E_6
7	x_7	y_7	E_7	7	8		E_7
8	x_8	y_8	E_8	8	1		E_8

Wireframe cube

To represent a cube defined by eight vertices and 12 edges, one needs the following tables :



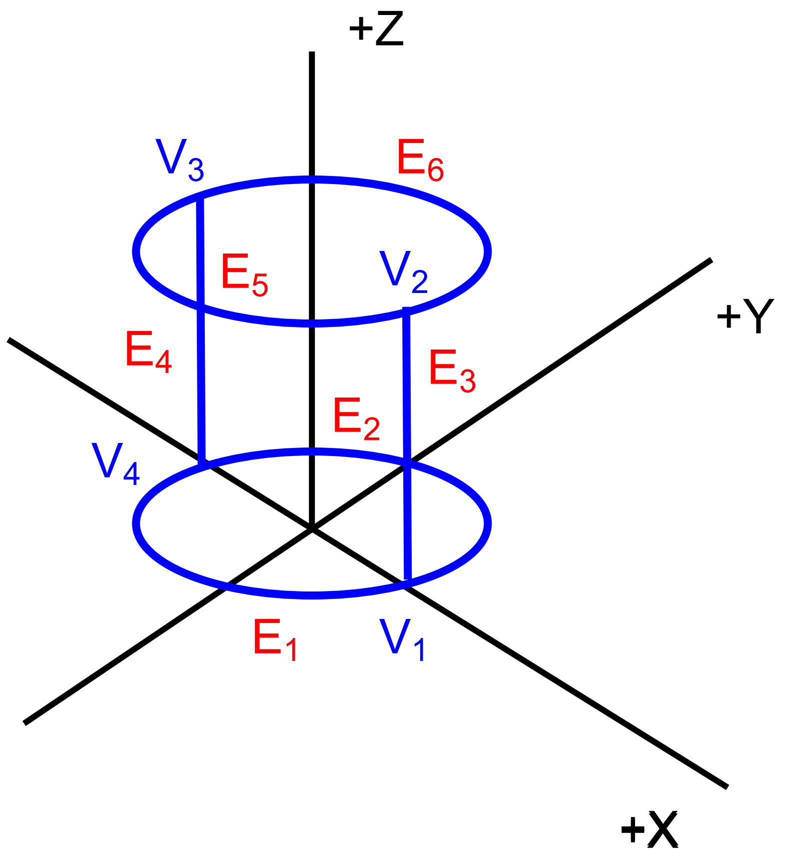
Vertex Table			
Vertex #	x	y	z
1	1	1	1
2	1	-1	1
3	-1	-1	1
4	-1	1	1
5	1	1	-1
6	1	-1	-1
7	-1	-1	-1
8	-1	1	-1

Edge Table		
Edge #	Start Vertex	End Vertex
1	1	2
2	2	3
3	3	4
4	4	1
5	5	6
6	6	7
7	7	8
8	8	5
9	1	5
10	2	6
11	3	7
12	4	8

Wireframe Cylinder

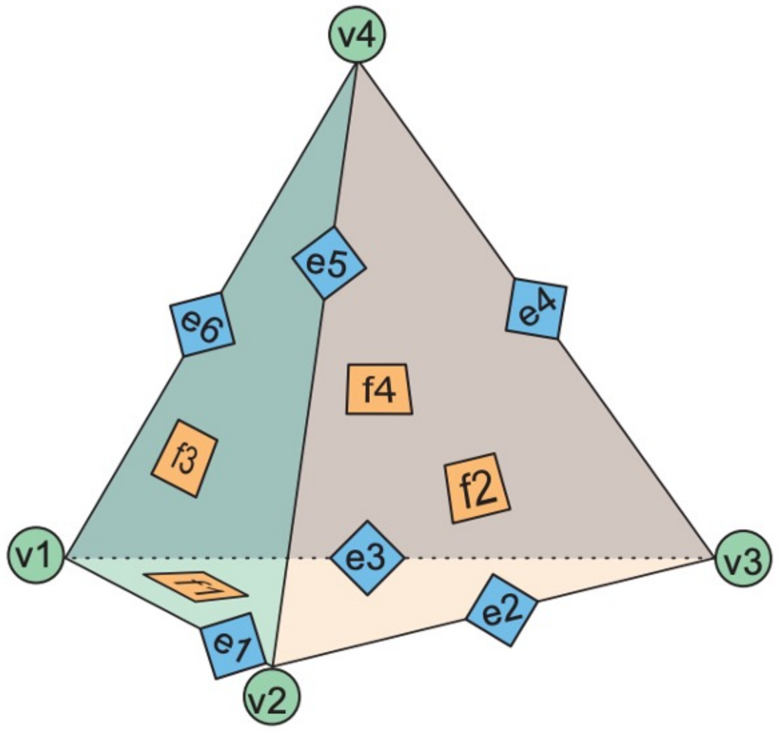
In the example show, the model definition (edges, vertices) was chosen to meet a validity criteria set.

- Three edges intersect at a vertex
- Edges delimited by two vertices



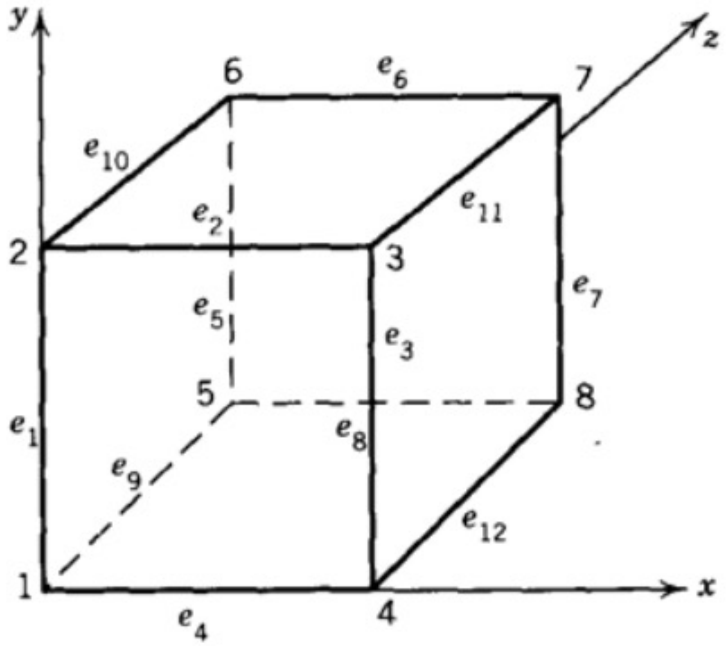
Vertex-edge-face graph

This system of relationships defines the topology of the modeled body, and can be described with the help of a graph, which is known as the **vertex-edge-face** (=vef) graph.



Solid	Faces	Face	Edges	Vertex	Coordinate	Edge	Vertices
1	1,2,3,4	1	1,2,3	1	2,0,0	1	1,2
		2	2,4,5	2	0,0,0	2	2,3
		3	1,5,6	3	3,0,0	3	3,1
		4	3,4,6	4	1,1,3	4	3,4
						5	2,4
						6	1,4

Unit cube



Faces	Edges	Vertices
1	1,2,3,4	1,2,3,4
2	1,9,5,10	1,5,6,2
3	8,7,6,5	5,8,7,6
4	3,11,7,12	3,7,8,4
5	4,9,8,12	1,5,8,4
6	2,10,6,11	2,6,7,3

Vertex (8x3)

	x	*y*	*z*
1	0.0	0.0	0.0
2	0.0	1.0	0.0
3	1.0	1.0	0.0
4	1.0	0.0	0.0
5	0.0	0.0	1.0
6	0.0	1.0	1.0
7	1.0	1.0	1.0
8	1.0	0.0	1.0

Edges (12x2)

	Vertices	
1	1	2
2	2	3
3	3	4
4	4	1
5	5	6
6	6	7
7	7	8
8	8	5
9	1	5
10	2	6
11	3	7
12	4	8

Face (6x4)

	Edges			
1	1	2	3	4
2	1	9	5	10
3	8	7	6	5
4	3	11	7	12
5	4	9	8	12
6	2	10	6	11

Face (6x4)

	Vertices			
1	1	2	3	4
2	1	5	6	2
3	5	8	7	6
4	3	7	8	4
5	1	5	8	4
6	2	6	7	3

Vertex connectivity matrix

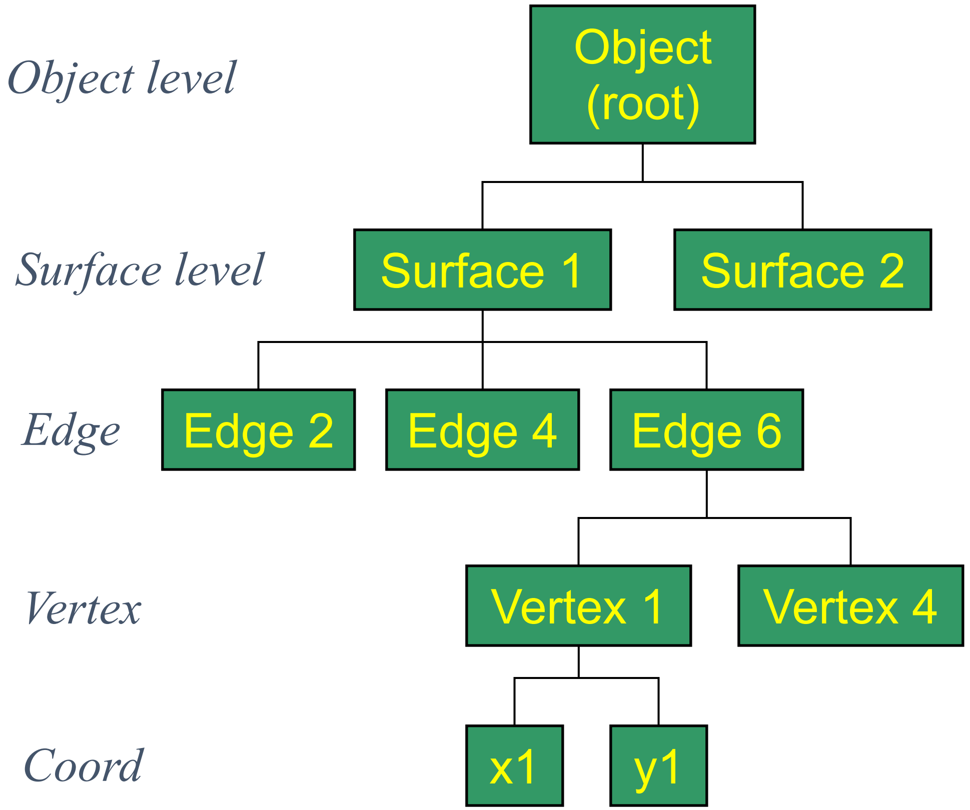
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
1	0	1	0	1	1	0	0	0
2	1	0	1	0	0	1	0	0
3	0	1	0	1	0	0	1	0
4	1	0	1	0	0	0	0	1
5	1	0	0	0	0	1	0	0
6	0	1	0	0	1	0	1	0
7	0	0	1	0	0	1	0	1
8	0	0	0	1	1	0	1	0

Face connectivity matrix

	1	*2*	*3*	*4*	*5*	*6*
1	0	1	0	1	1	1
2	1	0	1	0	1	1
3	0	1	0	1	1	1
4	1	0	1	0	1	1
5	1	1	1	1	0	0
6	1	1	1	1	0	0

Hierarchical database

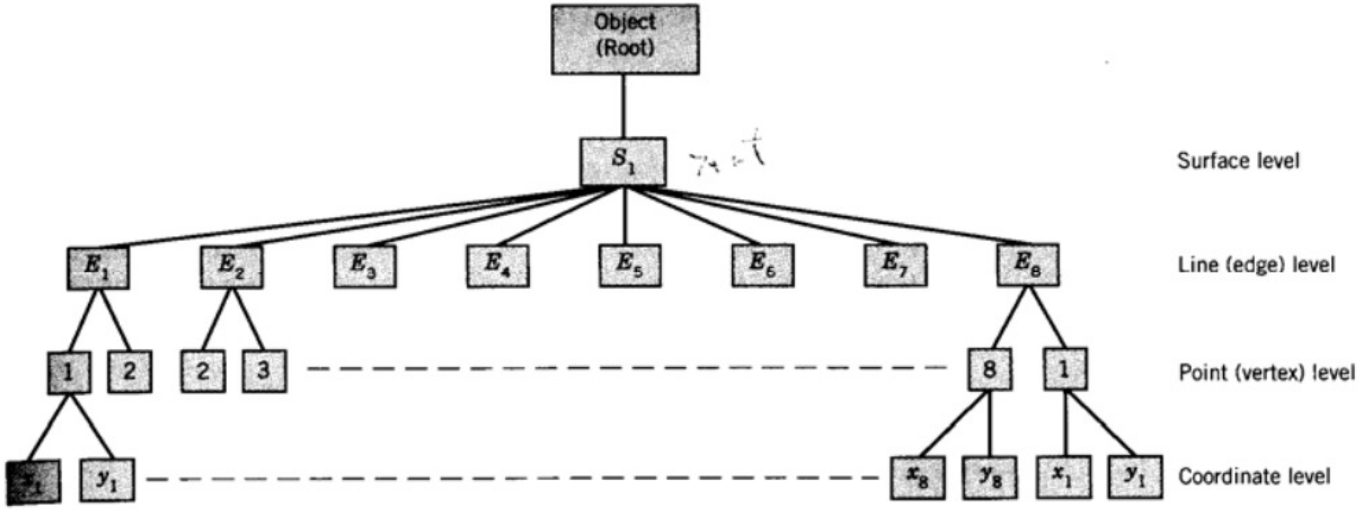
- Relational database
  - A set of lists, uses arrays for storage
- Hierarchical database
  - A trees structure, think of a company's executive structure
- Network database
  - Use data pointers



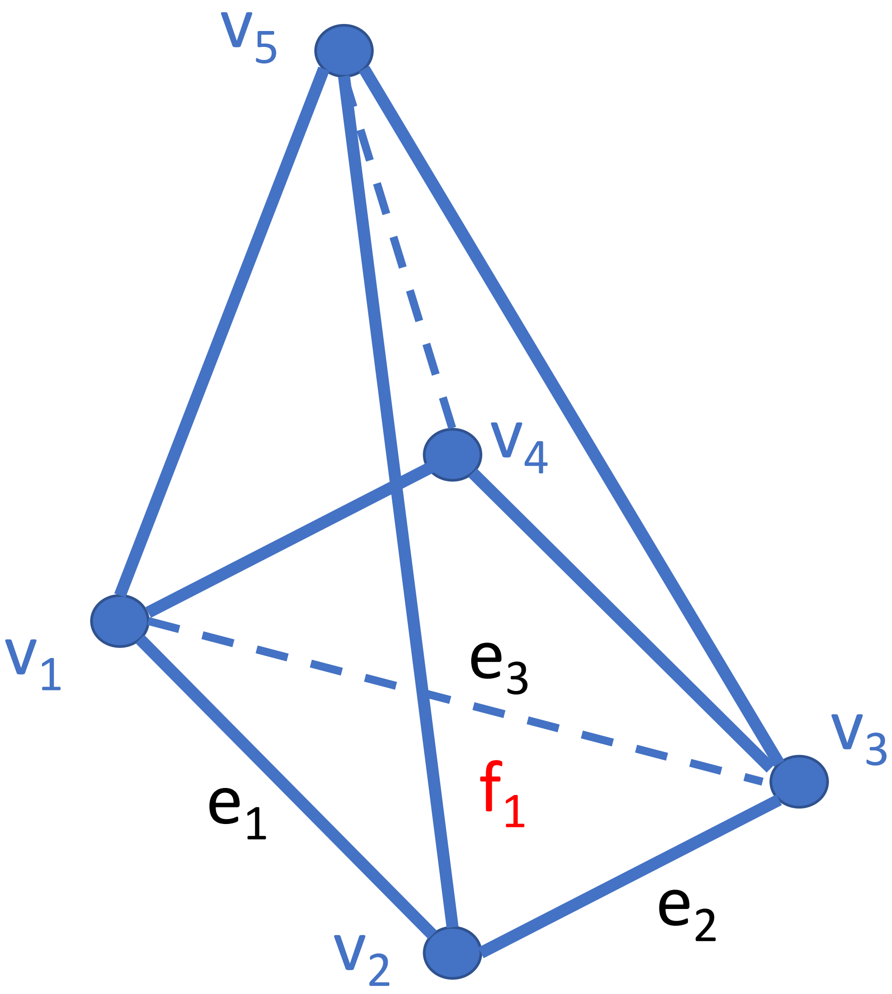
Hierarchical database

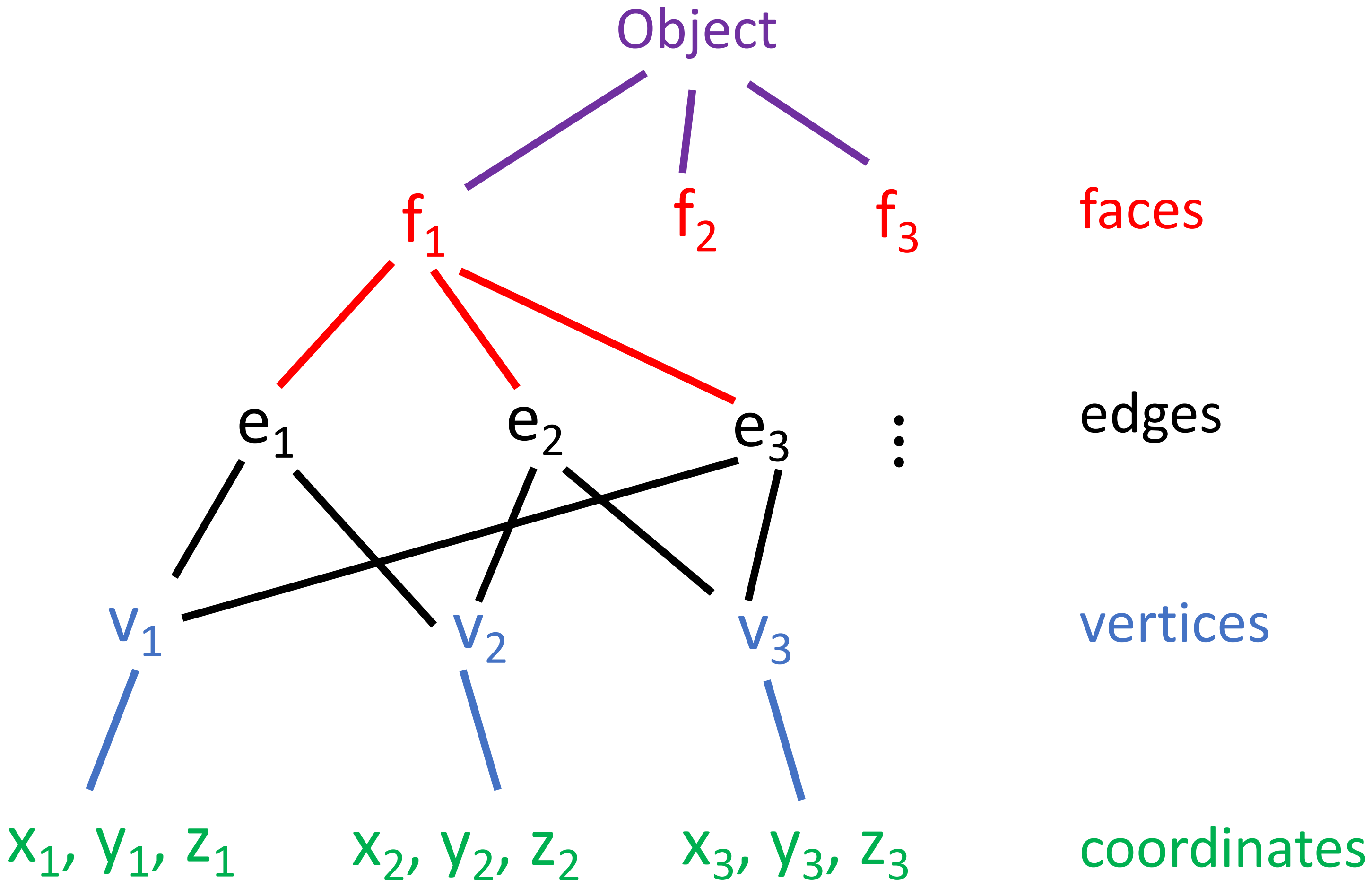
Hierarchical database is a tree structure composed of a hierarchy of elements called nodes. Hierarchical models are usually simple and fast, but only few relations in the real world are purely hierarchical.

The other disadvantages of hierarchical structure are the hierarchical implementation usually creates redundancy and a danger of inconsistency (i.e. cannot implement non-manifold models)

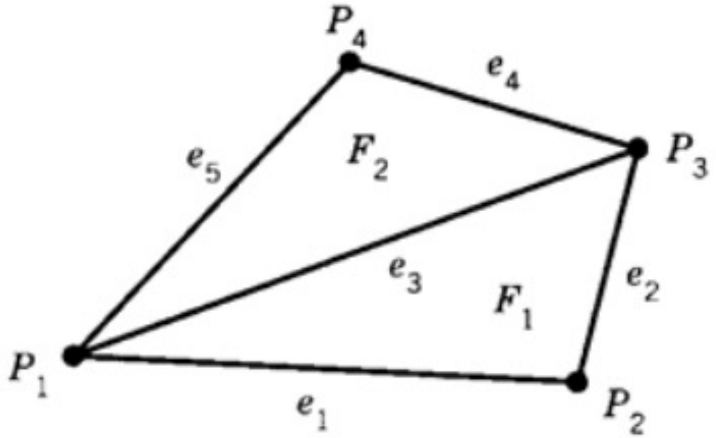


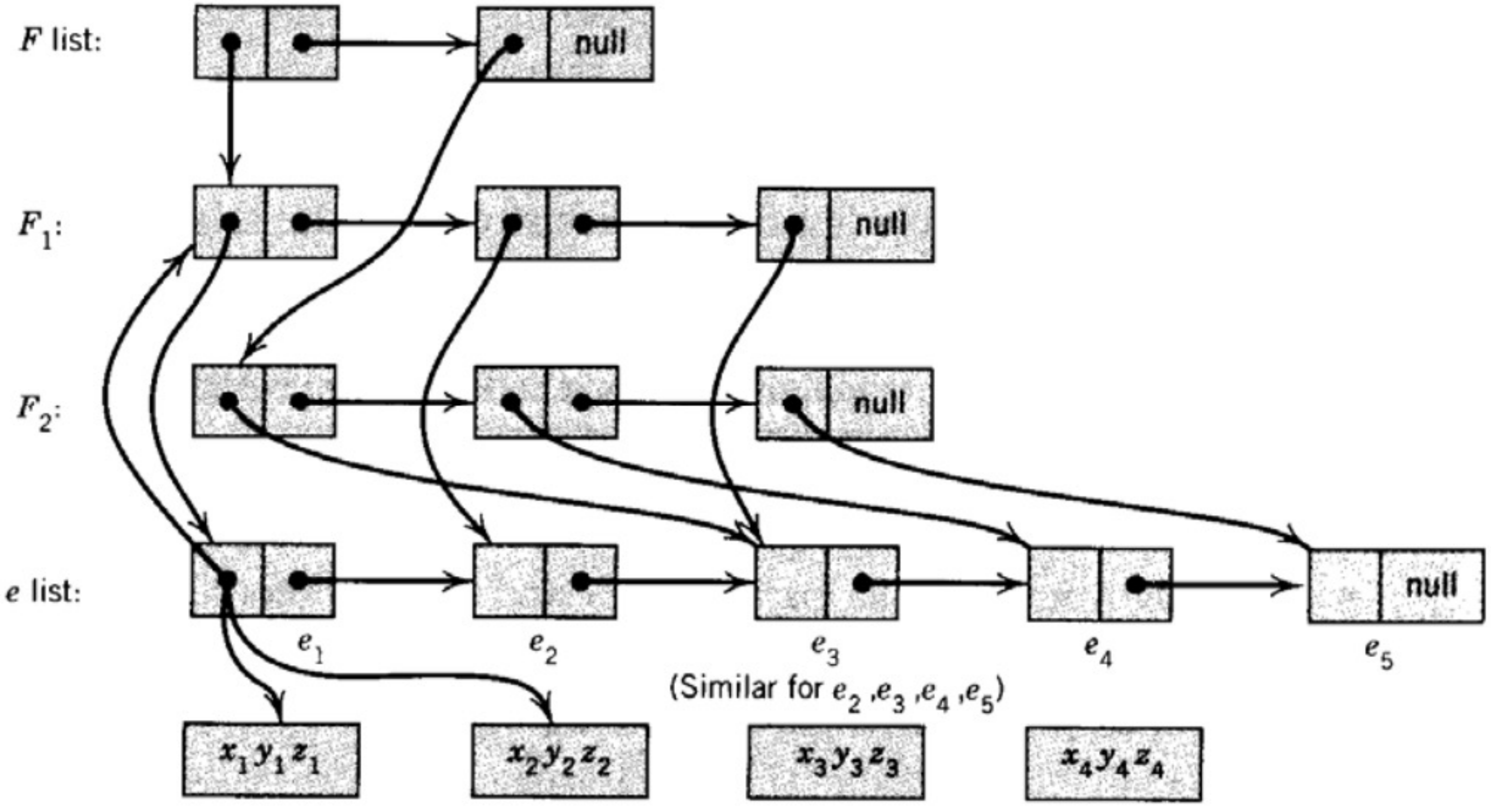
Hierarchical database



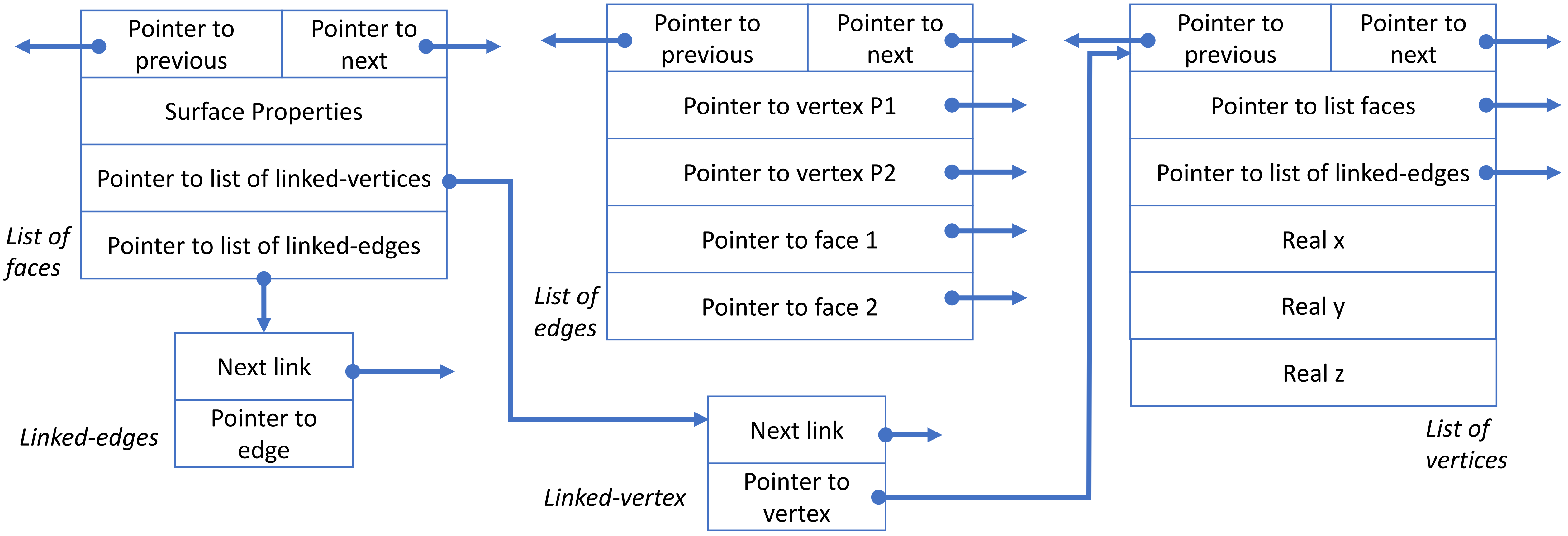


Hierarchical database





General hierarchical link data structure



Network database

- Relational database
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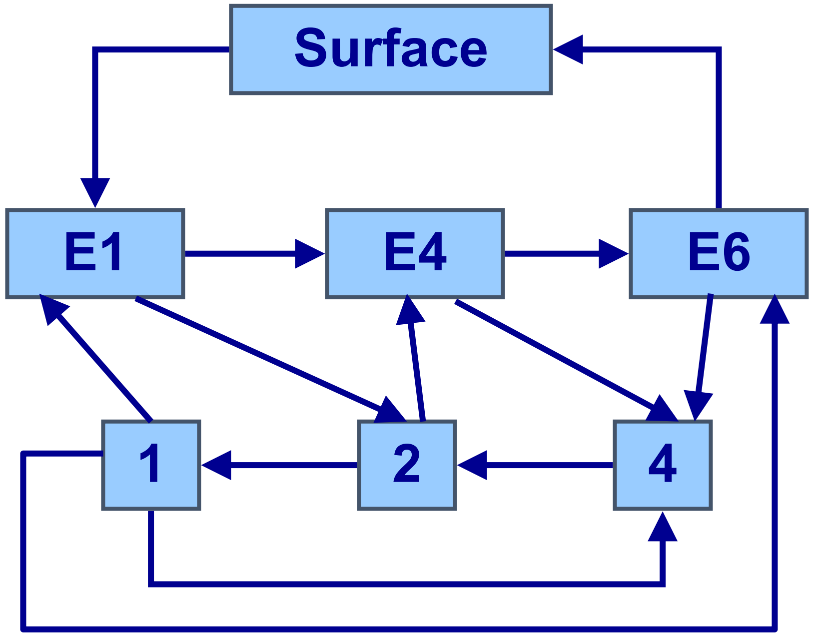
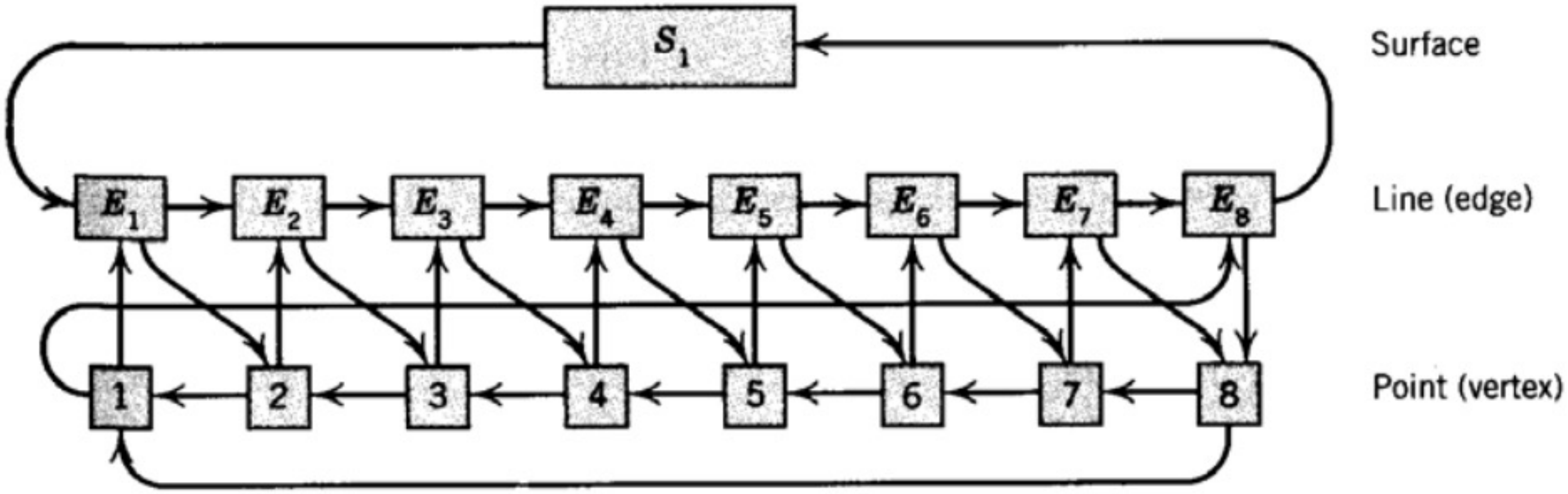
  - Use data pointers

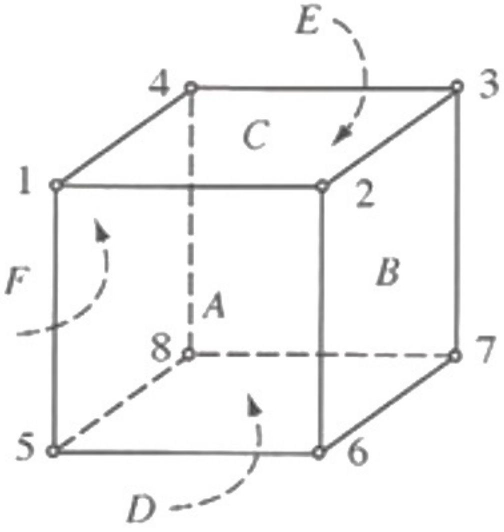


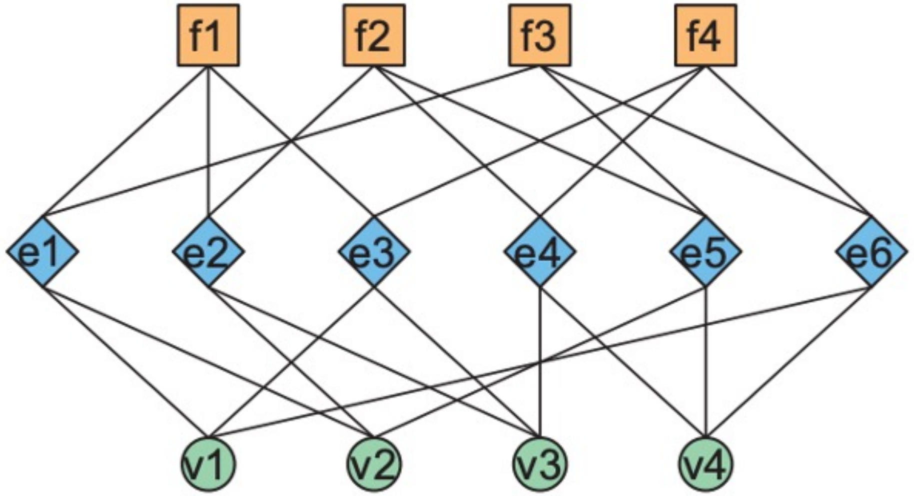
Network database

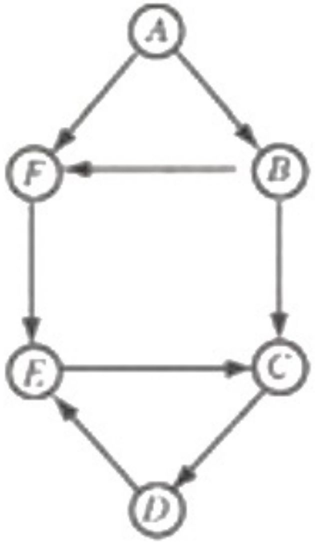
Network database has a “many-to-many” relationship among its elements; elements at each level can be connected to many elements of the level above.



Network database







Solid	Faces	Face	Edges	Vertex	Coordinate	Edge	Vertices
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		2	2,4,5	2	0,0,0	2	2,3
		3	1,5,6	3	3,0,0	3	3,1
		4	3,4,6	4	1,1,3	4	3,4
						5	2,4
						6	1,4

Real object & wireframe model

In most cases, 3D wire-frames are used to model objects in the real world, providing (amongst other things) a tool to aid object visualization.

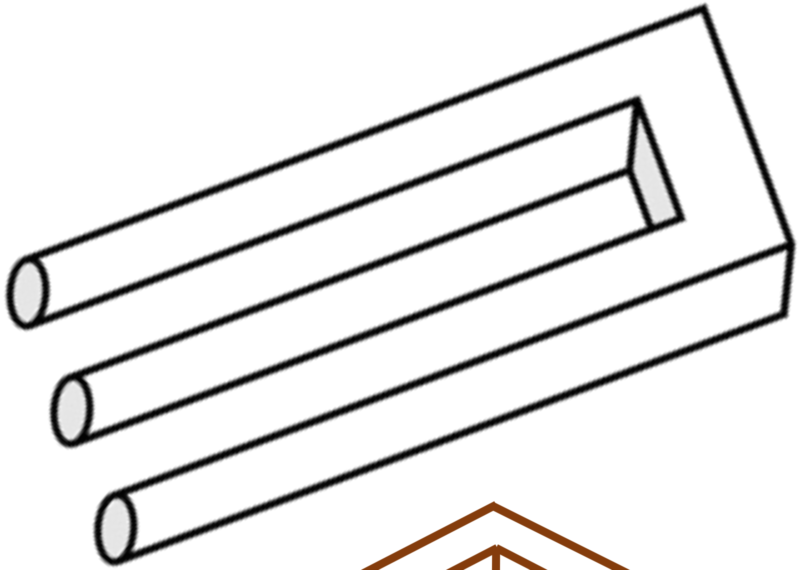
All of requirements are satisfied, three-dimensional wire-frames still have failings in two major areas:

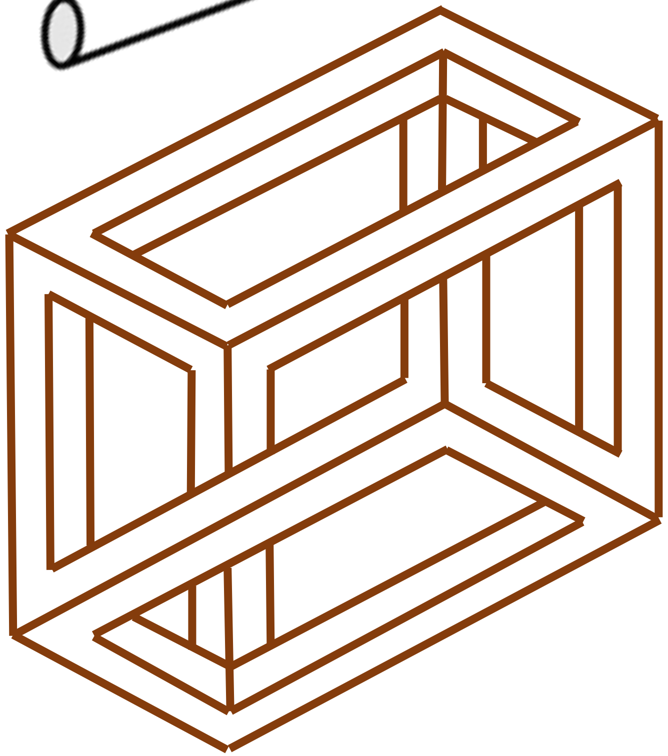
- Lack of validity
- Ambiguity

Impossible (nonsense) object

This can best is to select a represented by the type of structure, consisting of simple (vertex) points and linear edges.

- Each point is well defined in 3D space;
- Each edge is associated with just two end-points;
- The face edges all form closed loops;
- No faces are self-intersecting.





Line labelling

Impossible object can be represented by a regular 2D line drawing, as it cannot be reconstructed into a valid 3D object because the object does not exist.

The original purpose of line labelling was as a method of identifying and rejecting impossible drawings.

Line labelling concept has also been used to validate whether the 3D object which has been represented by a 2D line drawing is a possible or impossible object.

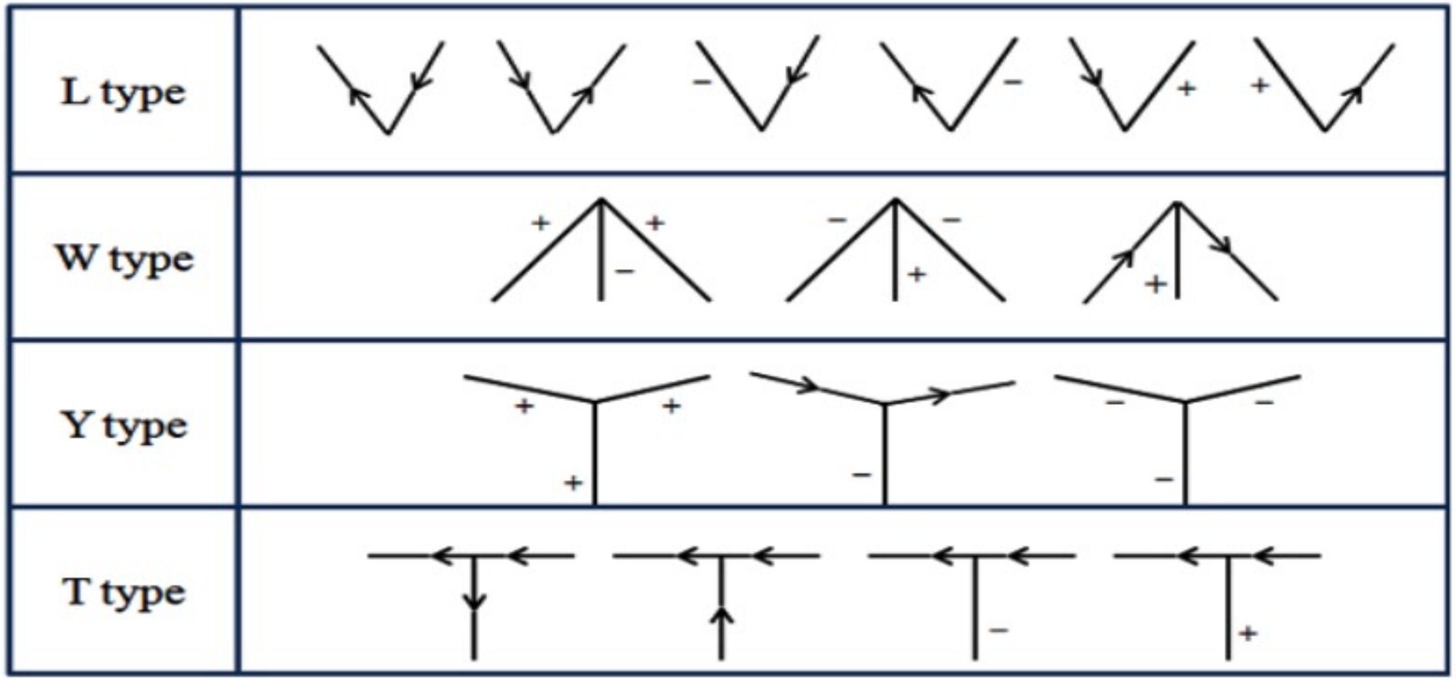
Huffman-Clowes-Waltz line labelling

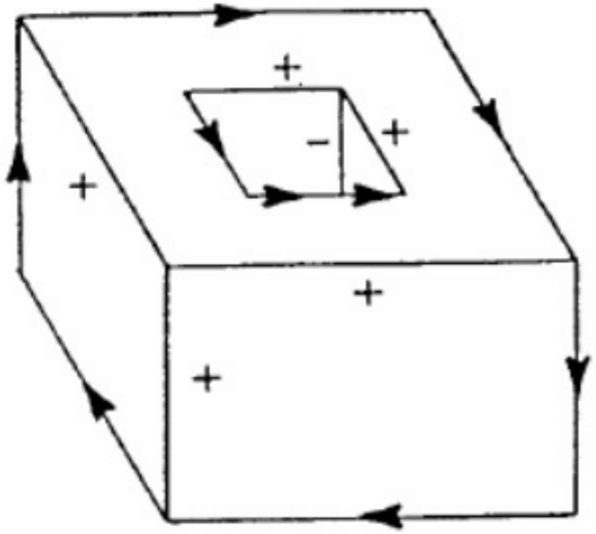
Line labelling is a method for interpreting 3D structure of a 2D line drawing, in which labels are placed on lines to indicate their relative position in space and their convexity.

The works on line labelling have been carry out since the early 1970’s, started by D.A.Huffman. Initially, it is used to interpret trihedral of planar objects as highlighted by Huffman and Clowes. The years after, the convention has already been extended by D.I. Waltz and it is known as Huffman-Clowes-Waltz line labelling.

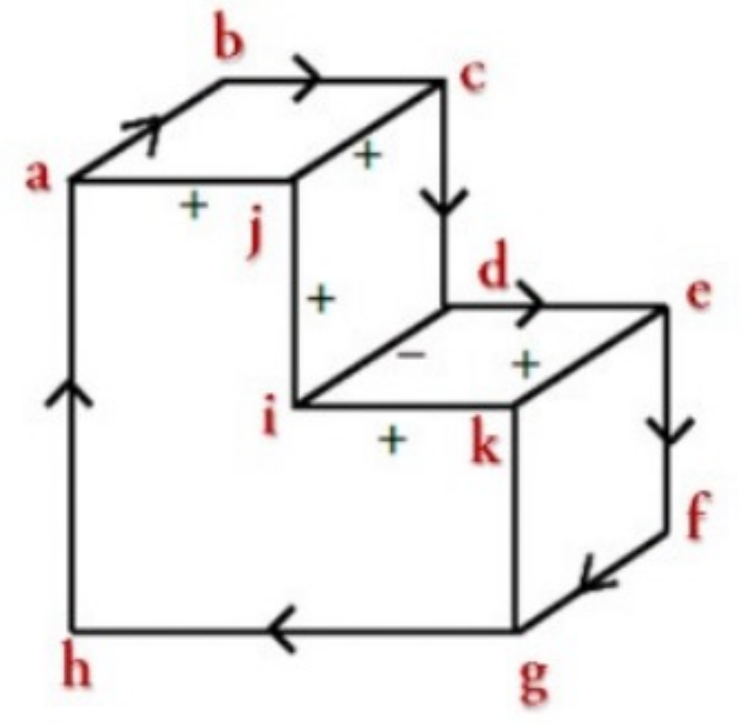
D.A. Huffman, Impossible objects as nonsense sentences, In M16, 1971
M.B. Clowes, On seeing things, Artificial Intelligence 2, 1 (1971), pp. 79-116.
D.I. Waltz, Generating semantic descriptions from drawings of scenes with shadows, PhD disstertaion, AI Lab MIT (1972).

Huffman-Clowes-Waltz line labelling

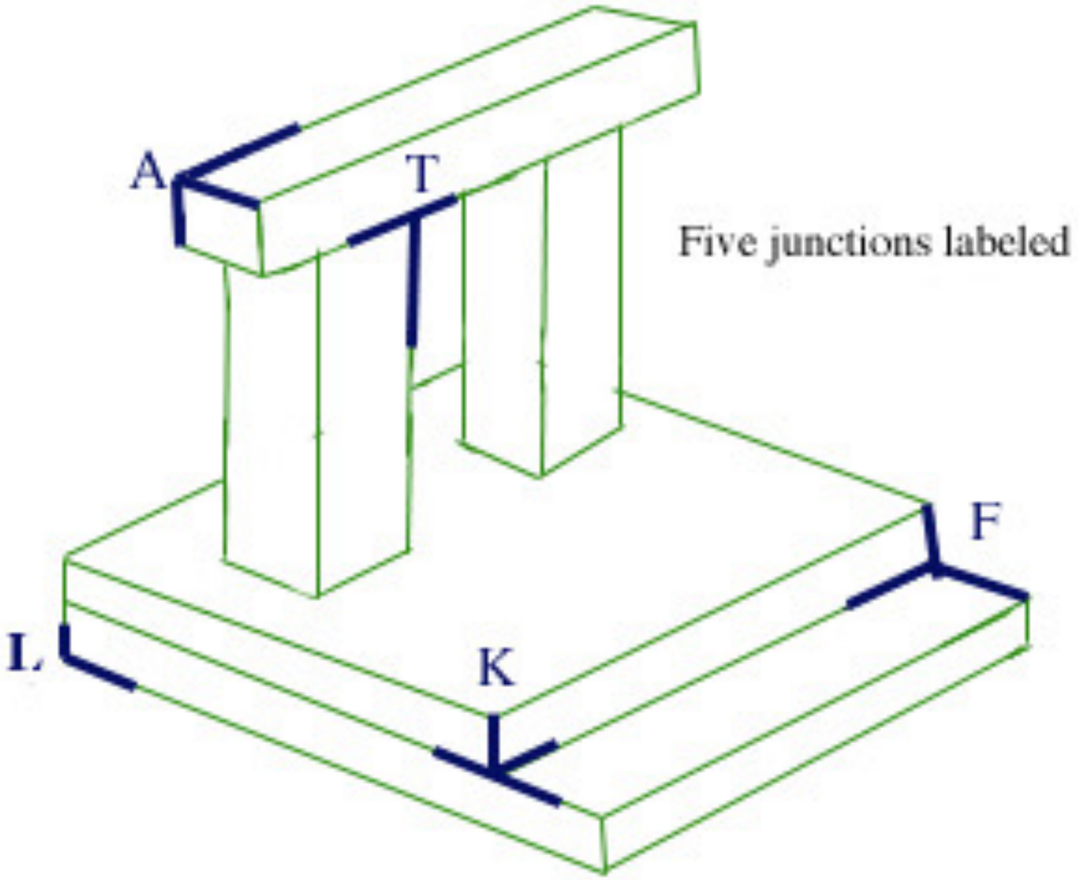


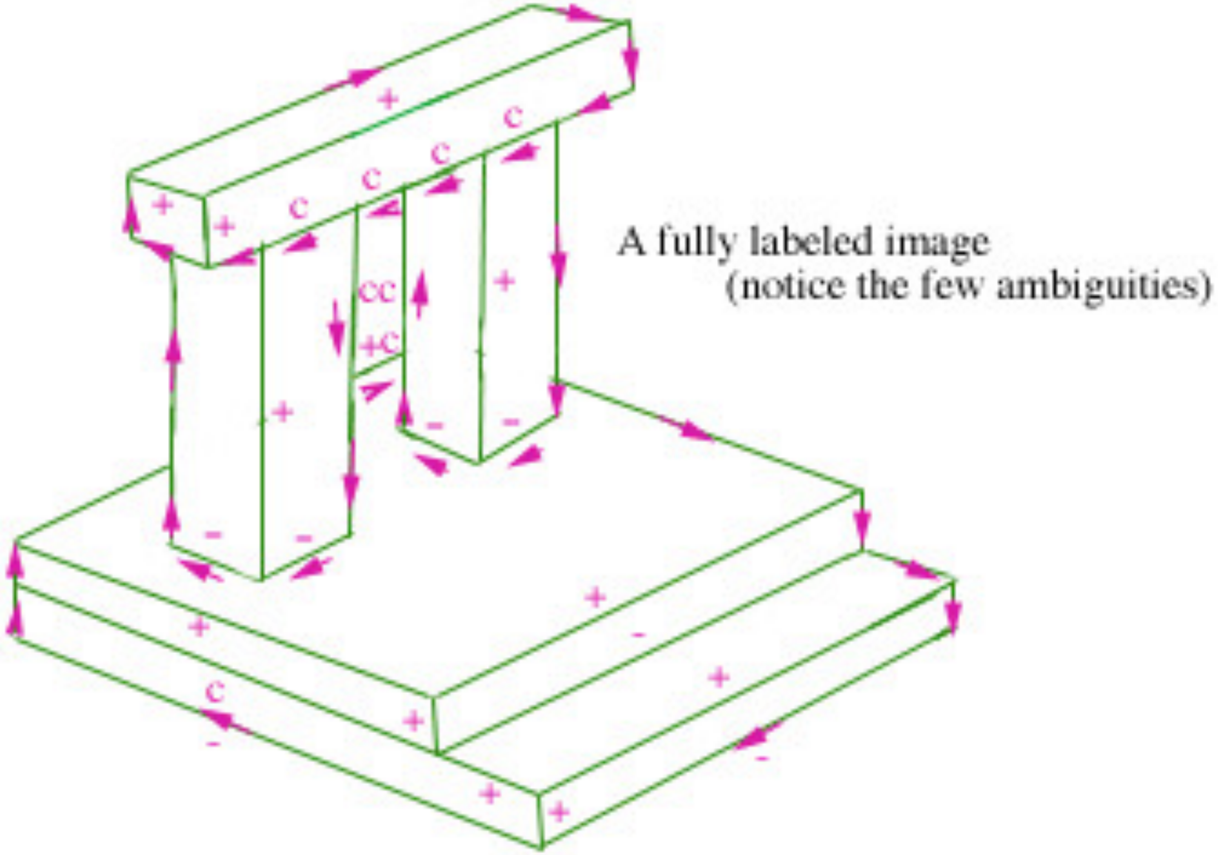


Example

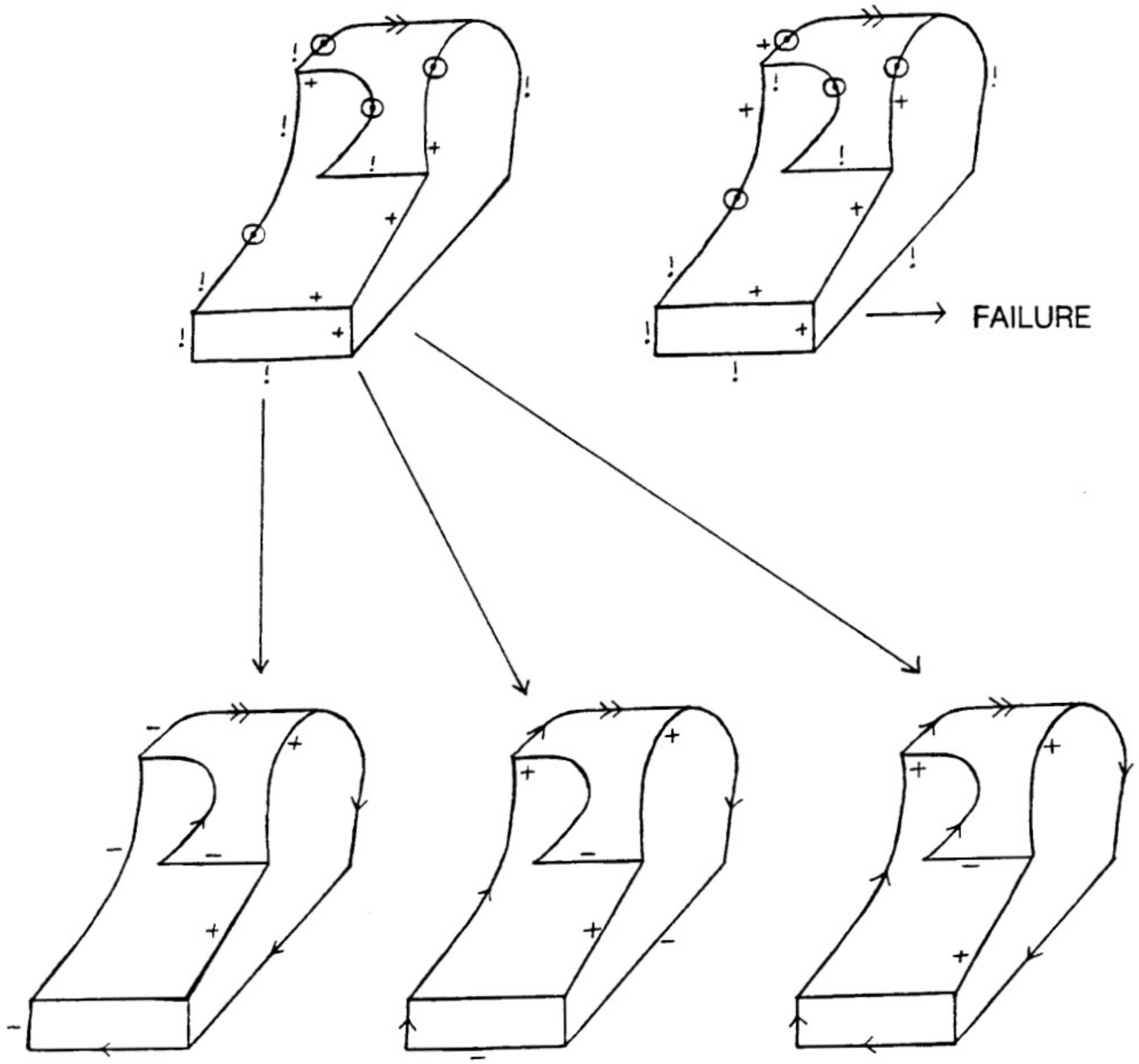
2D line drawing for a 3D solid object L-block	Line labelling scheme	Example of the labelled line
	L-type	$\{(abc),(bcd),(fgh)\}$
	W-type	$\{(hbj,a),(dfk,e),(jkd,i)\}$
	Y-type	$\{(ice,d),(ica,j),(gie,k)\}$
	T-type	-

Huffman-Clowes-Waltz line labelling





Line labelling for curved objects



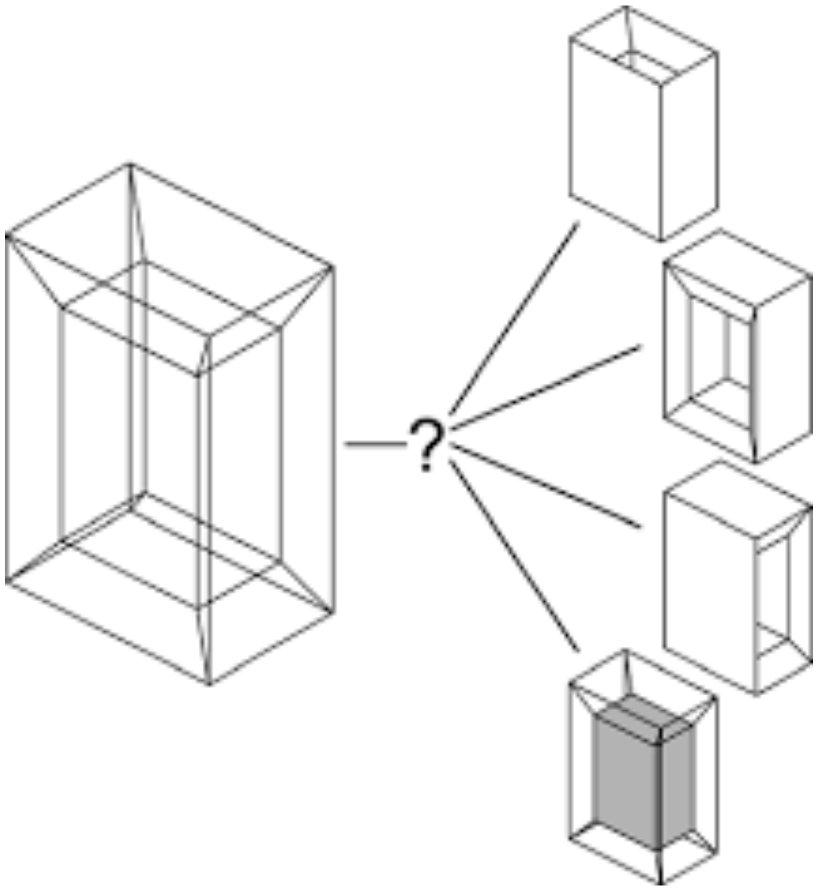
J. Malik, Interpreting line drawings of curved objects, International Journal of Computer Vision, 1 (1987), pp.73-103.

Ambiguity

While wireframe uses the simplest data structures, it is ambiguous. The following is a well-known example that consists of 16 vertices and 32 edges.

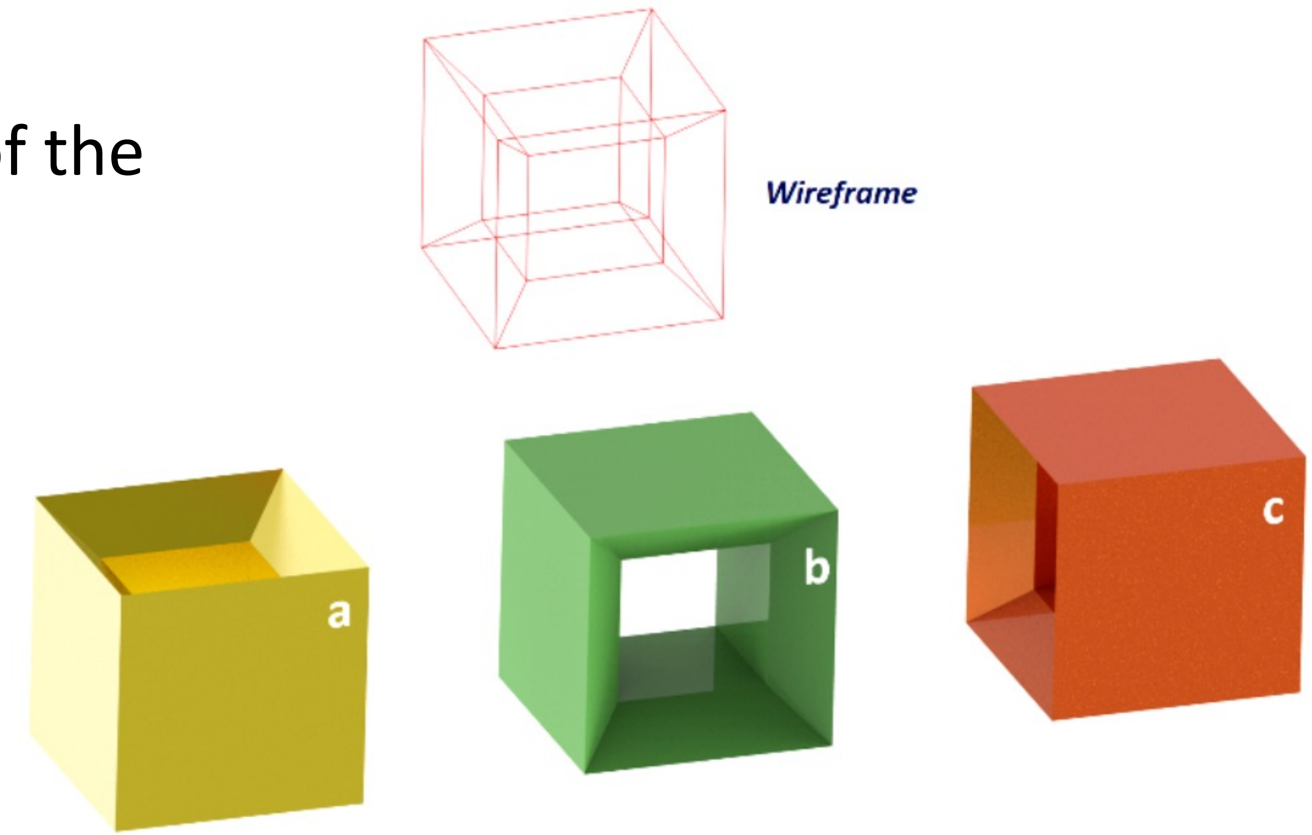
We know it represents a solid and each of the quadrilaterals (some of them are squares) defines a face of the solid.

Because of wireframe models are ambiguous, their uses are limited.



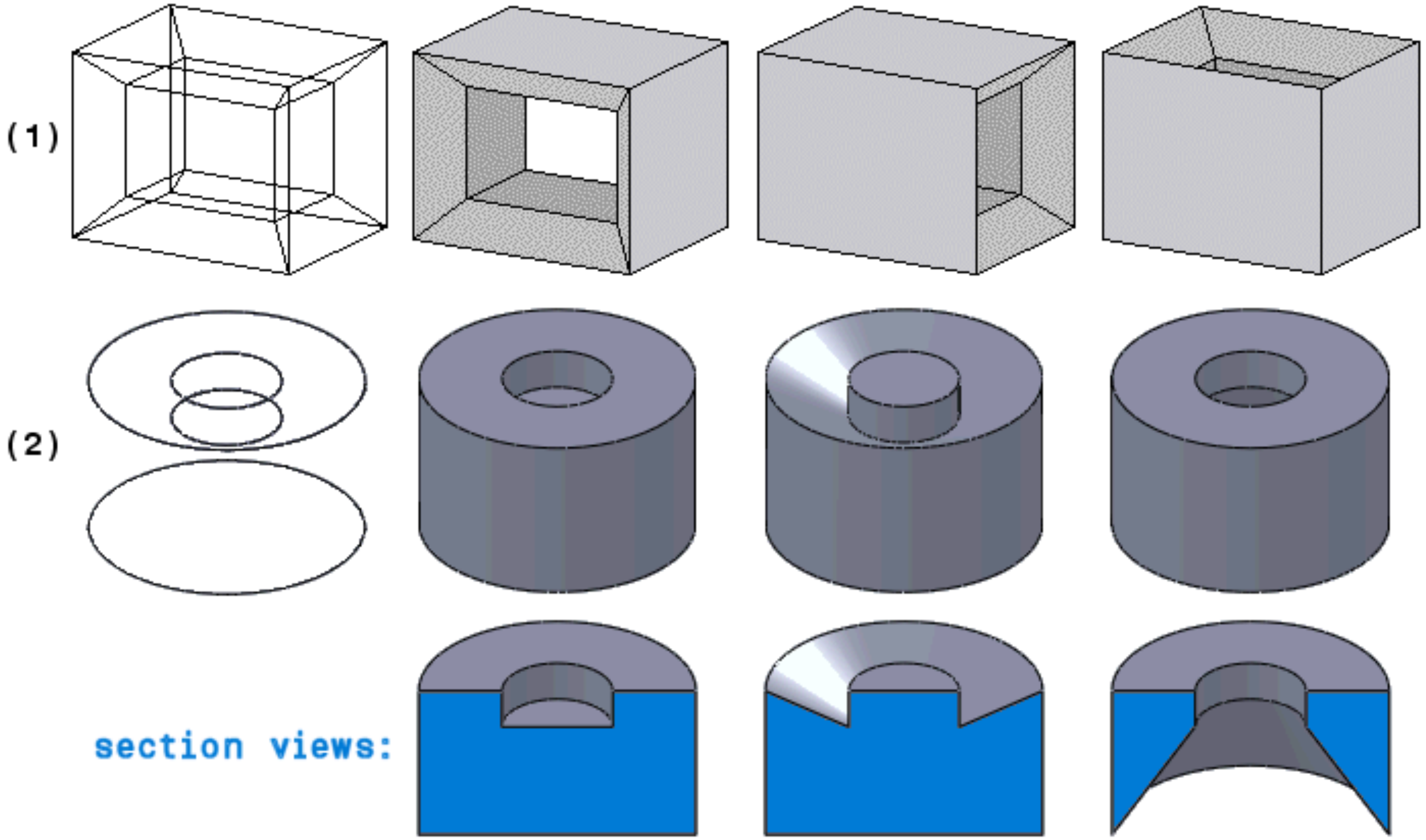
Ambiguity

There is no information about the inside and outside boundary surfaces of the object. Thus it is not possible to unambiguously derive the faces of the object from its edges or calculate properties like mass or volume necessary for further analysis



Ambiguity

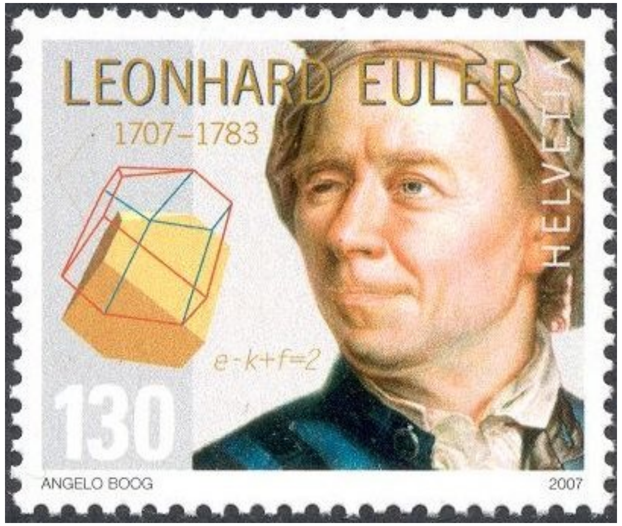
The wire-frame can represent any of the other real objects shown in the figure.



Validity Checking for Simple Solids

Euler (1752) a Swiss mathematician proved that polyhedra that are homomorphic to a sphere are topologically valid if they satisfy the equation.

This formula encapsulates a fundamental property of those three-dimensional solids we call *polyhedra*, which have fascinated mathematicians for over 4000 years.



Leonard Euler
(1707-1783)

Euler's polyhedron formula

It appears to have been the French mathematician Legendre who gave the first proof using Spherical Geometry.

S.A.J. L'huilier found a slight generalization of Euler's formula to take into account polyhedra having holes.

The formula was generalized to n-dimensional polytopes by Schlafli and proved by Poincare.

Adrian Marie Legendre
(1752-1833)

Simone L'huilier
(1750-1840)

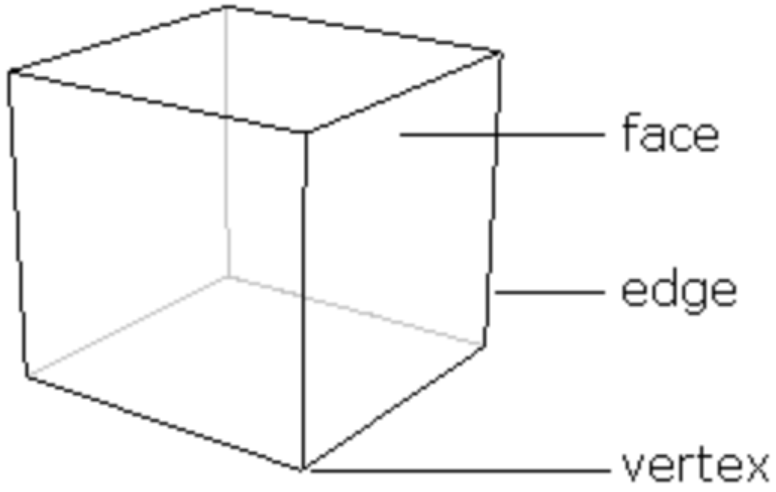
Euler's polyhedron formula

Defines an invariant relationship among the vertices, edges, and face loops of a polyhedron.

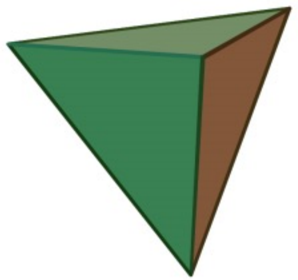
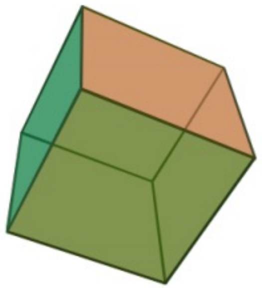
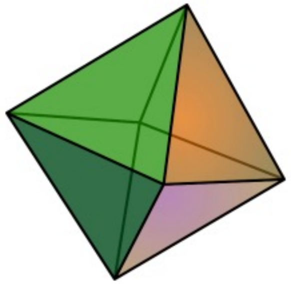
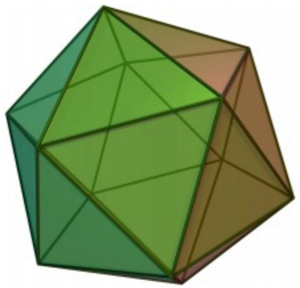
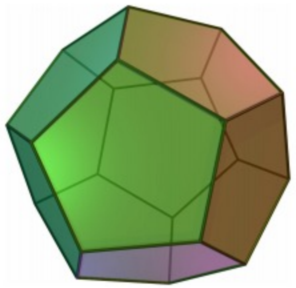
Valid for simple polyhedra (continuous, no hole)

$$V - E + F = 2$$

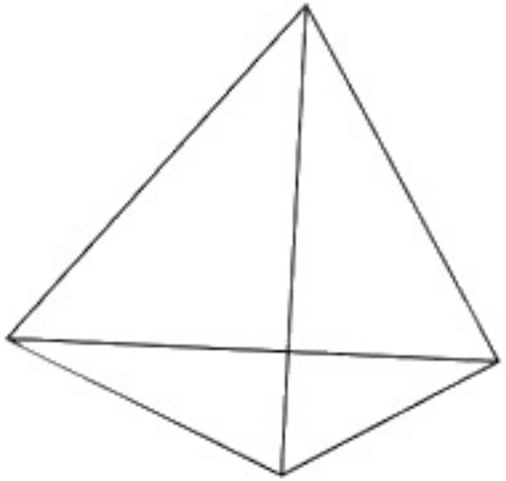
V (# vertices)
E (# edges)
F (# face loops)



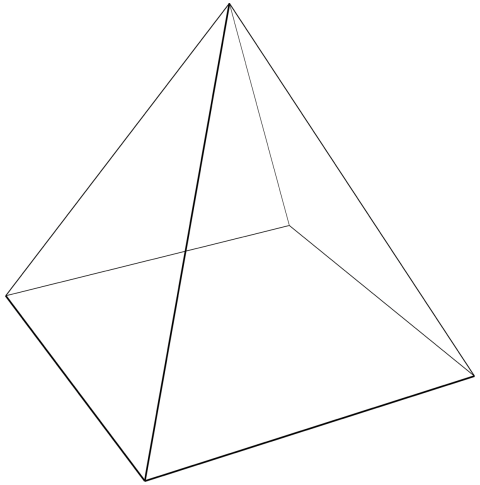
Platonic solids

				
Tetrahedron	Cube	Octahedron	Icosahedron	Dodecahedron
$v = 4$ $e = 6$ $f = 4$	$v = 8$ $e = 12$ $f = 6$	$v = 6$ $e = 12$ $f = 8$	$v = 12$ $e = 30$ $f = 20$	$v = 20$ $e = 30$ $f = 12$

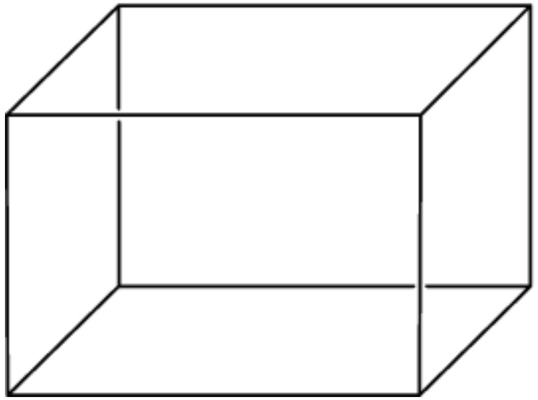
Euler's polyhedron formula



$V = 4$
$E = 6$
$F = 4$
$4 - 6 + 4 = 2$

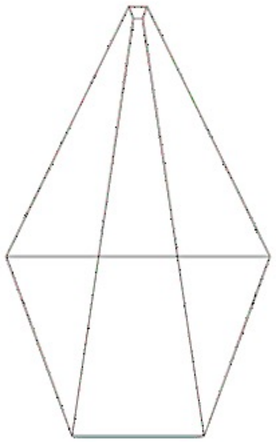


$V = 5$
$E = 8$
$F = 5$
$5 - 8 + 5 = 2$

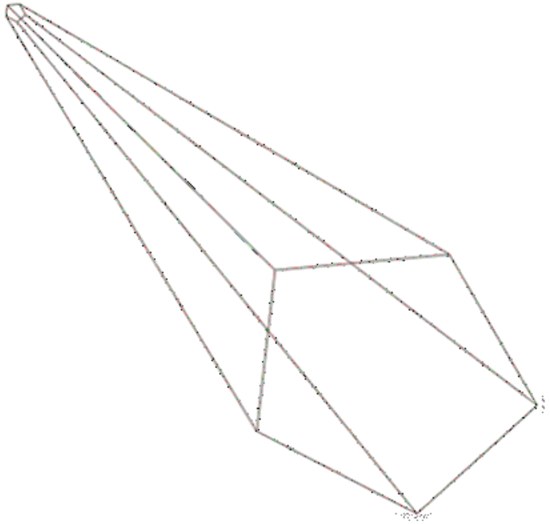


$V = 8$
$E = 12$
$F = 6$
$6 - 12 + 8 = 2$

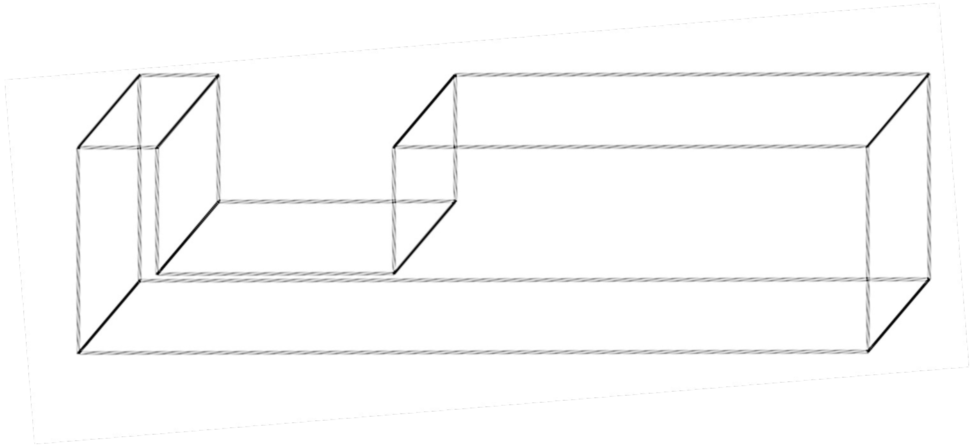
Euler's polyhedron formula



$V = 5$
$E = 8$
$F = 5$
$5 - 8 + 5 = 2$

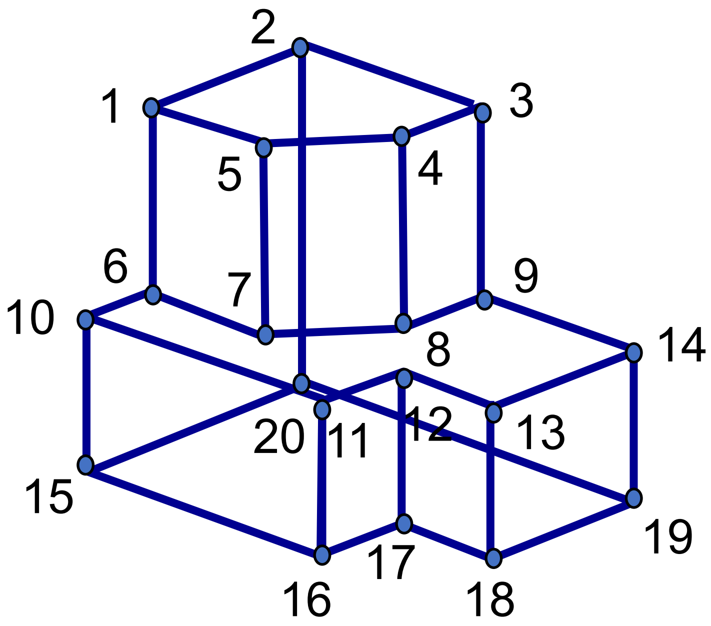


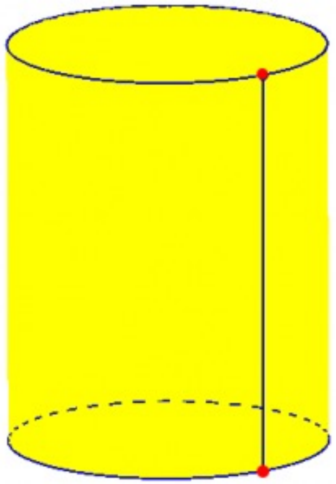
$V = 6$
$E = 10$
$F = 6$
$6 - 10 + 6 = 2$



$V = 16$
$E = 24$
$F = 10$
$10 - 24 + 16 = 2$

Euler's polyhedron formula





$V = 20$
$E = 30$
$F = 12$
$20 - 30 + 12 = 2$

$V = 2$
$E = 3$
$F = 3$
$2 - 3 + 3 = 2$

Expanded Euler's polyhedron formula

For evaluating complex polyhedrons formulation is expanded to include:

– hole loops, (any loop which is completely enclosed within another)

– through holes or genus (a feature that completely penetrates the object adds to its genus, no penetrating features, genus = 0)

– shells (sets of faces which bound a volume, either space or void)

Euler-Poincaré formula

The Euler-Poincaré formula describes the relationship of the number of vertices, the number of edges and the number of faces of a manifold.

It has been generalized to include potholes and holes that penetrate the solid.

Euler Poincare formula

This is known as the Euler-Poincare Law:

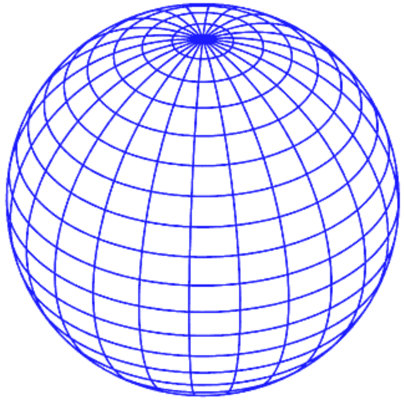
$$V - E + F - L = 2(S - G)$$

V = # of vertices
E= # of edges
F = # of faces
L = # of hole loops (sometimes "H" for hole)
S = # of shell bodies (sometimes "C")
G = # of thru holes, genus, passage features

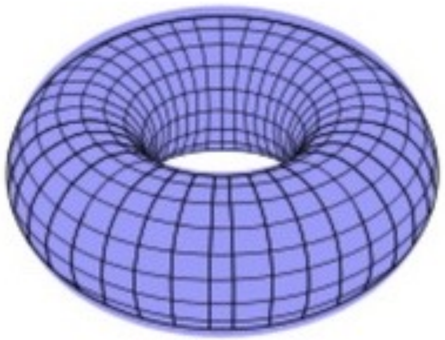
Henri Poincaré
(1854-1912)

Loops (rings), Genus & Bodies

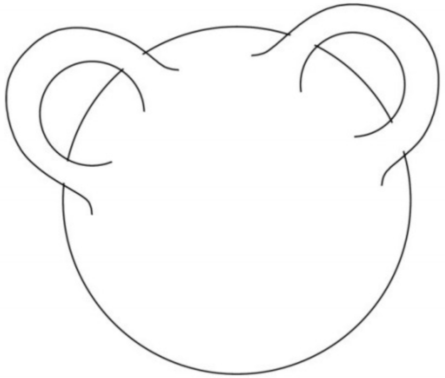
Genus zero



Genus one

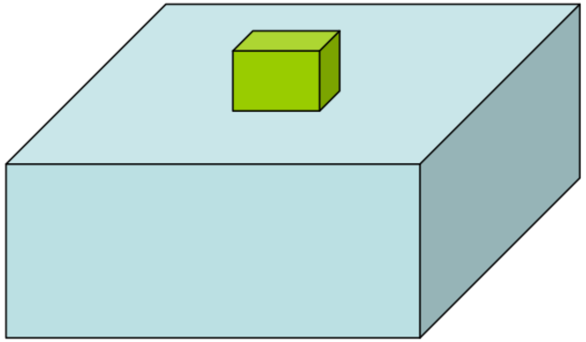


Genus two

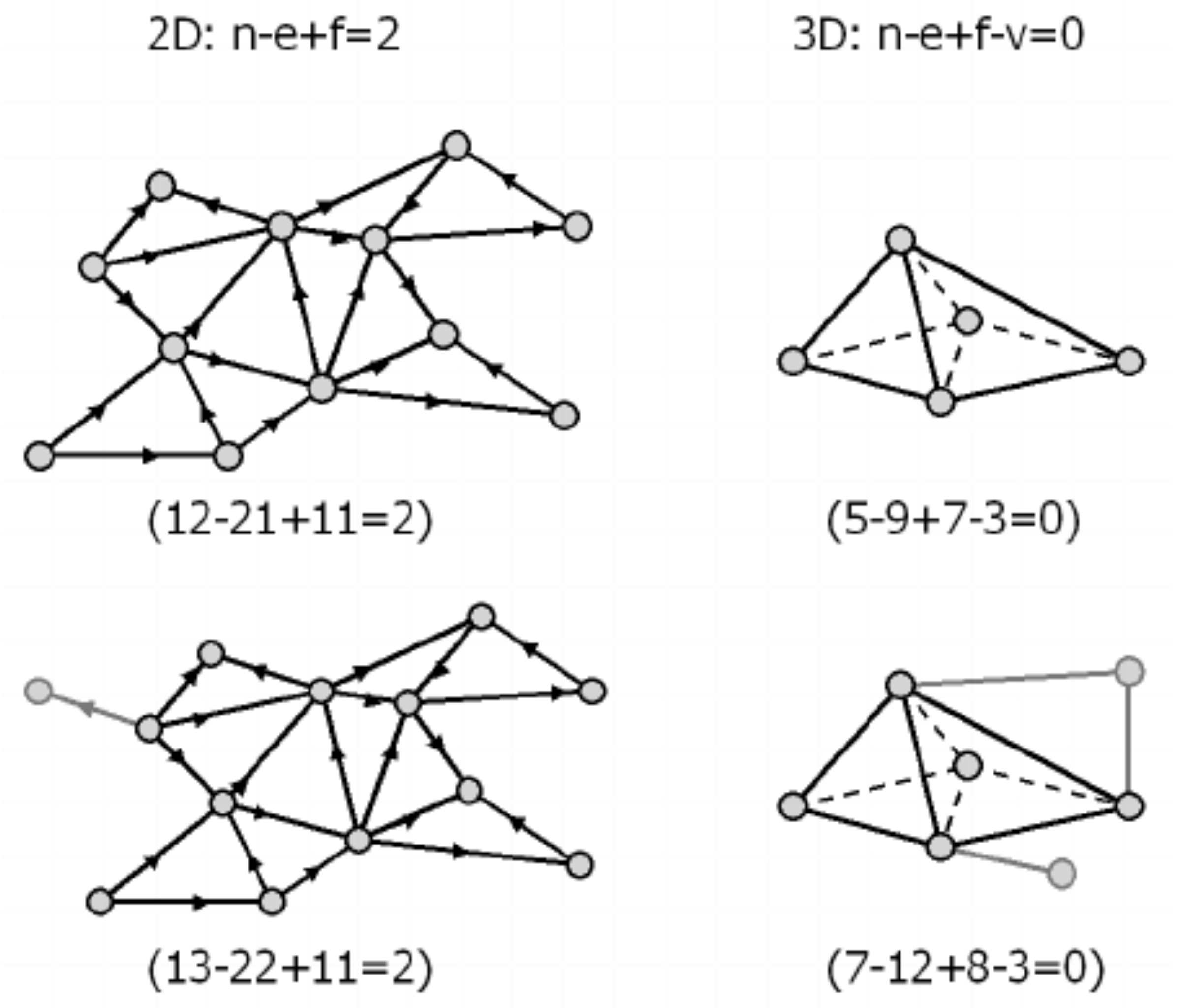


Genus three

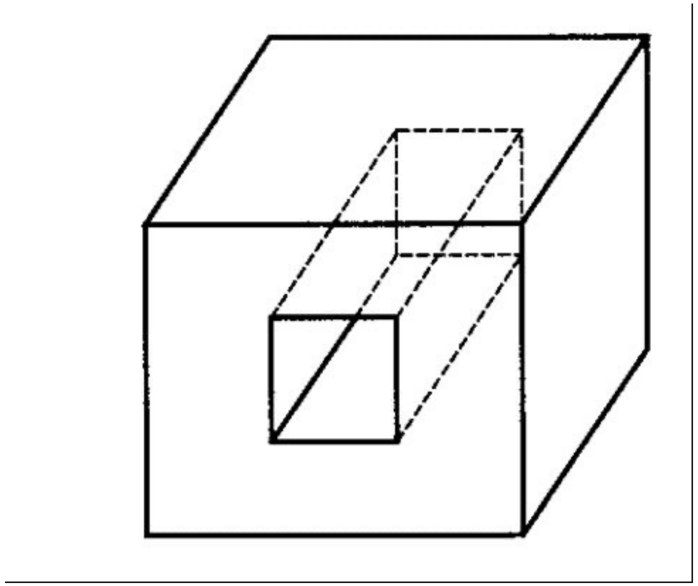
One inner loop

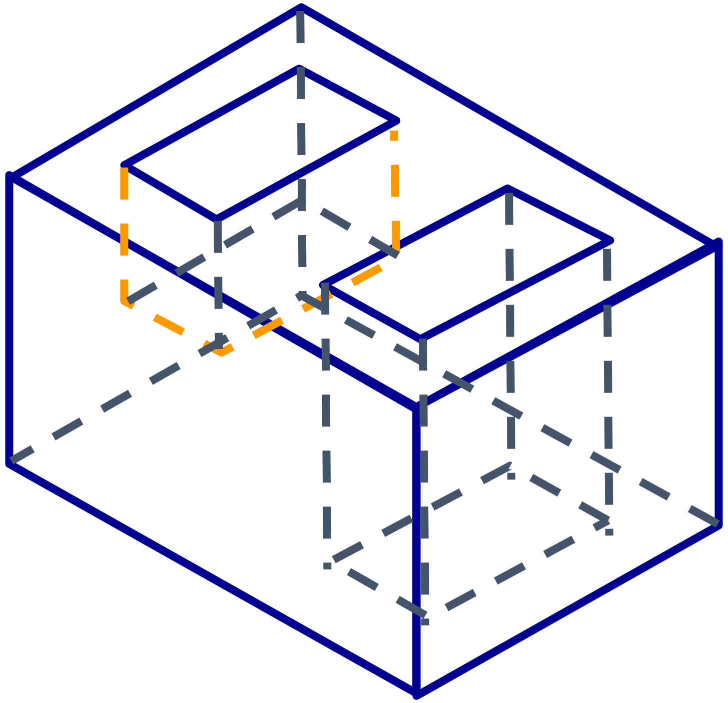


Euler Poincare formula



Euler-Poincare Formula



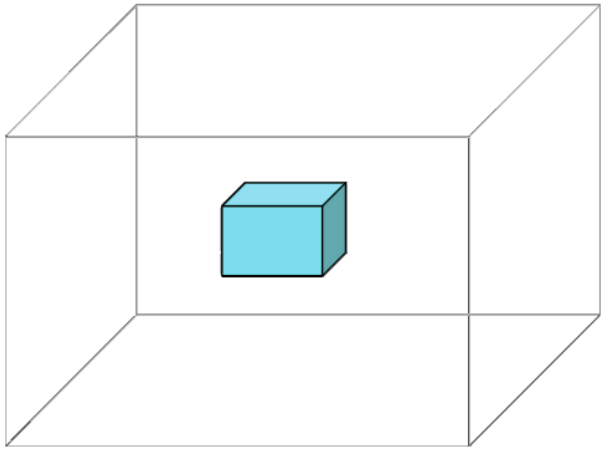


$$V - E + F - L = 2(S - G)$$
$$16 - 24 + 10 - 2 = 2(1 - 1)$$

$$V - E + F - L = 2(S - G)$$
$$24 - 36 + 15 - 3 = 2(1 - 1)$$

Polyhedra with not through holes

INTERIAL HOLE (VOID)



$V = 16$

$E = 24$

$F = 12$

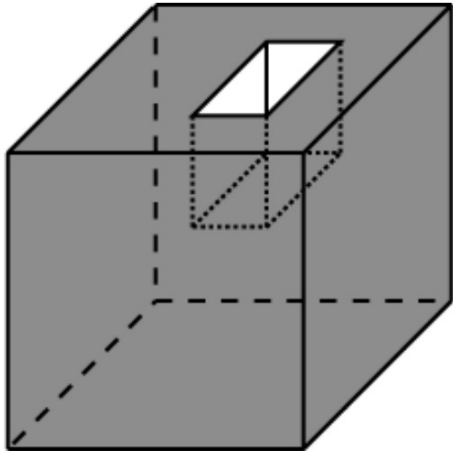
$L = 0$

$B = 2$

$G = 0$

$16 - 24 + 12 - 0 = 2(2 - 0)$

SURFACE HOLE



$V = 16$

$E = 24$

$F = 11$

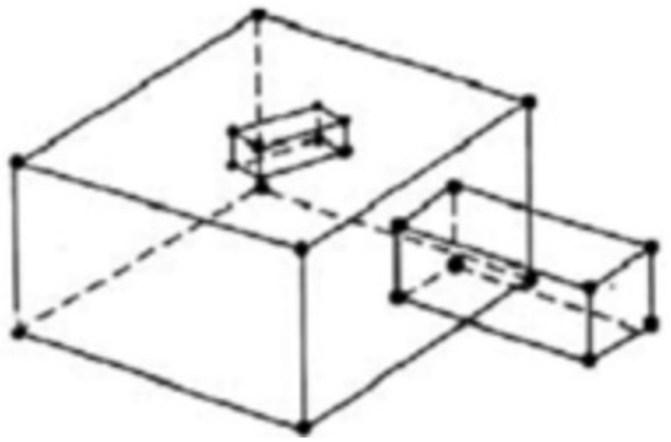
$L = 1$

$B = 1$

$G = 0$

$16 - 24 + 11 - 1 = 2(1 - 0)$

Polyhedra with face of inner loop



These are same to the first with the exception that a face might be bounded by more than one loop of edges.

$V = 24$

$E = 36$

$F = 16$

$L = 2$

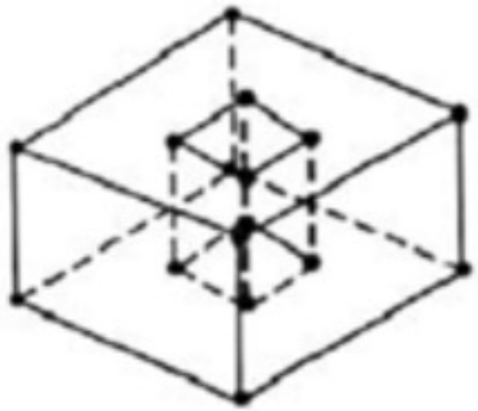
$B = 1$

$G = 0$

$24 - 36 + 16 - 2 = 2(1 - 0)$

Polyhedra with through holes (handles)

THROUGH HOLE



$V = 16$ $B = 1$

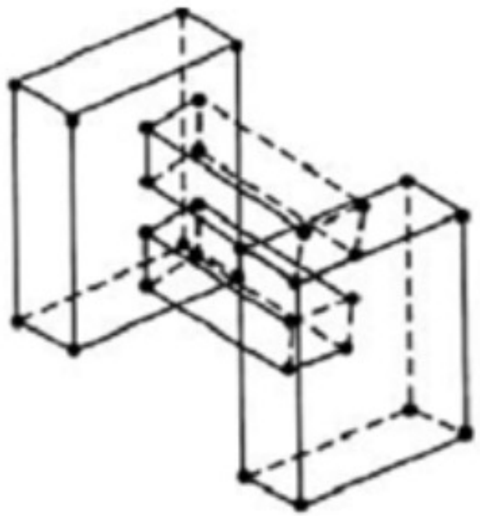
$E = 24$ $G = 1$

$F = 10$

$L = 2$

$16 - 24 + 10 - 2 = 2(1 - 1)$

HANDLES/THROUGH HOLE



$V = 32$ $B = 1$

$E = 48$ $G = 1$

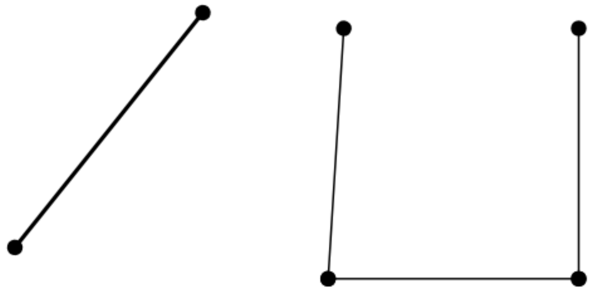
$F = 20$

$L = 4$

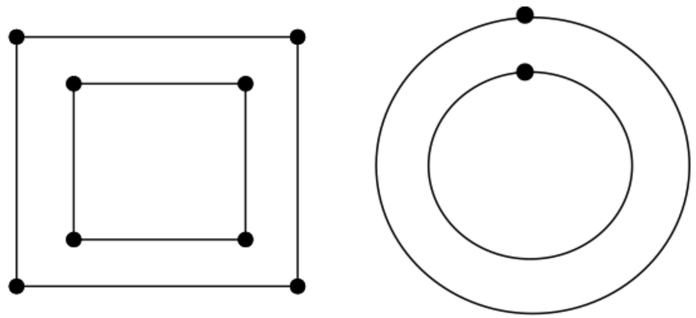
$32 - 48 + 20 - 4 = 2(1 - 1)$

Validity Checking for open objects

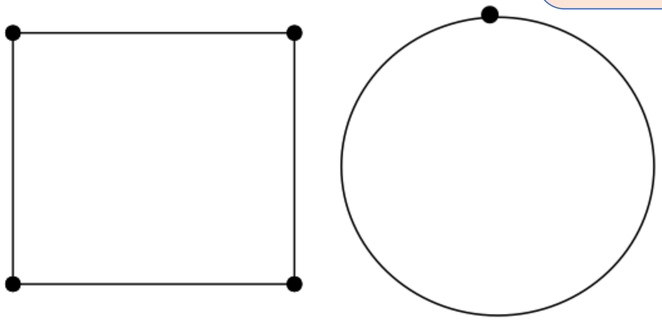
Wireframe polyhedra



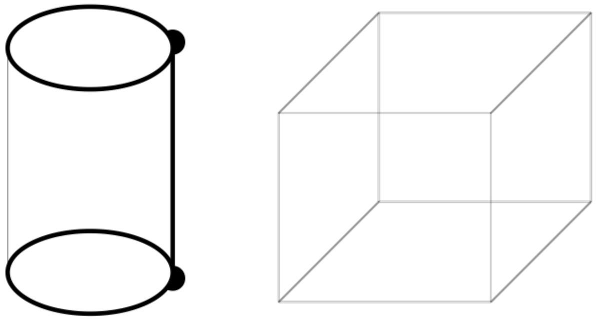
Shell polyhedra



$$V - E + F - L = S - G$$



Lamina polyhedra



Open 3D polyhedra

Abrrivations

CAGD	Computer Aided Geometric Design
HRL	Hidden line removal
QH	Quantitatice hiding